POWER CONCEPT OF THE STABILITY

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Abstract:- *Some „weak points" of contemporary stability concepts are discussed in this contribution. As shown, the power conception can lead to the correct results in cases where classical approaches failed. Essential conclusions of so-called fluctuation theory of stability are presented. Typical examples are given that to use common stability criteria, some unstable electrical systems must be tested carefully.*

Introduction

Contemporary methods of the stability testing of electrical systems utilize so-called vector description that have been assumed from the Newton's dynamics. The decision about the system stability (or the instability) is based on the *time evolution of the system variables* after the system being deflected from its equilibrium state by arbitrary fluctuation. If the system comes back to the original equilibrium state after the fluctuation becomes extinct, this state is declared as stable. In opposite case, we talk about the instability, the boundary of stability, etc. However, we show that this vector description is only the description of the consequences and it can be insufficient for the understanding of real causes of the instability.

Let us consider circuit on Fig. 1. Analyzing DC operating point (i.e. disconnected capacitor) by the arbitrary method based on the Kirchhoff's laws, we obtain paradox nodal voltages (see Fig. 1). Our result is in conformity with the Kirchhoff's laws, but the state is certainly unstable and thus unobservable.

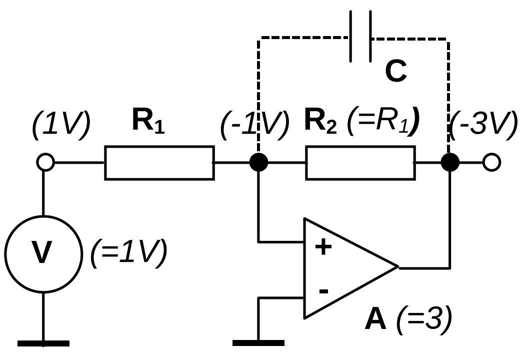

Fig. 1 Circuit where the "pole-location" method fails.

Now consider capacitor C that is connected in parallel to the feedback resistor R_2. Such circuit has single real pole, which is negative in spite of evident instability!

This example and many others promote the opinion that the vector system description has the phenomenological character. It reflects only the outer side of real processes, not their essence. This fact is obvious even from the ways of the stability definition. For the circuits linearized round their equilibrium points, classical definitions could be summarized as follows: *The equilibrium state is stable if the effect of the time-limited fluctuation which deflects the system from this state tends to zero.* This effect is measured by the circuit reaction that is some of the circuit variables. The measure is set by the outside observer (designer) if he chooses proper coordinates (for example nodal voltages).

It is known from the vector algebra that one function (signal) can be measured by the second one. As a result, we obtain so-called mutual energy between two signals. For example, let us evaluate the effect of the current fluctuation $i(t)$ flowing through the impedance Z during time interval T as the integral $\varepsilon = \int_T i(t) \cdot v(t) dt$ where $v(t)$ is the voltage across the impedance outlets. In this case, the effect of the current fluctuation is not measured only by the *value* of reaction (i.e. voltage v – view of the "outside observer") but by *new measure* through this reaction (inner "system view"). We pass from the superficial vector description to the scalar description that can disclose *causes* of the investigated processes. This advantage is gained due to the fact that we use measure of the system for its measurement. The instantaneous power is the kernel of this inner measure.

We built so-called fluctuation theory of stability [1] that is based on the scalar description. The main idea of this theory is as follows: The equilibrium state of arbitrary system is permanently disturbed by numerous fluctuations. For electrical systems, these fluctuations are modeled by the voltage and current sources that affect each node and each branch. The special method of stability testing of dc operating points consists in the finding of so-called virtual eigenvalues. These system eigenvalues are load impedances or admittances of the sources of fluctuations. From the physical point of view, the resistive system linearized round tested dc operating point behaves as the set of *N* independent loading imittances of *N* sources of fluctuations [1]. The generalization to the equilibrium states of the dynamical systems is possible [2].

It can be shown that there are two possible causes of instability: the negative resistances and the controlled sources. Eliminating the first possibility (negative resistances can be modeled by the controlled sources), there is necessary only the same number of the source of fluctuation as the number of controlled sources inside the system [2]. Only the character of "fluctuation loading imittances" is then decisive for the stability of the equilibrium state.

If the current reaction to the time-limited fluctuation (or the voltage reaction to the current fluctuation) has the character of damped function, the equilibrium state is stable according to the vector conception of stability. On the contrary, the equilibrium state is unstable.

Aforementioned reactions can be easy modeled by impulse responses: impulse response of given loading impedance/admittance is the voltage/current reaction to the current/voltage fluctuation. Thus the impulse responses of all loading imittances consist information about the stability unambiguously. We formulate statement about the stability of equilibrium state:

Let $h_i(t)$, $i=1,..N$ are the impulse responses of loading imittances of the sources of fluctuations. The corresponding equilibrium state is stable if and only if

$$\forall i \in \langle 1,..N \rangle : \lim_{t\to\infty} h_i(t) = 0 . \qquad (1)$$

Equation (1) is the mathematical expression of the vector conception of the stability. According to this conception, the effect of time-limited fluctuation has to drop to zero.

Note: If we model fluctuation of the voltage controlled source, this source has to be perturbed only by the voltage, not current source of the fluctuation. Voltage source of the fluctuation causes the current system response. Modeling the fluctuation of the current controlled source, the effect is opposite. Then the stability is analyzed from the impedance impulse response. For instance, the fluctuation of voltage amplifier on Fig. 1 cause the current effect to the output admittance.

Equation (1) corresponds to the well-known stability/instability conception that is descriptive and does not consist information about the phenomena inside the system.

Let us try to build the scalar description of the stability.

System is stable if it accomplishes to restore violated equilibrium between itself and the source of the fluctuation. The loading imittance of corresponding source of the fluctuation informs about the character of the energy exchange that accompanies the equilibrium violation. Mathematical description of such imittance is serious problem in case of unstable systems.

Physics of stable systems

For simplicity (but without loss of universality) let us assume loading admittance of the voltage source of fluctuation *v(t)*. The reaction is in the current response *i(t)*. The impulse response $g(t)$ is the reaction to the Dirac voltage impulse. For stable system according to definition (1), the Fourier integral $G(j\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$ exists. This integral shows the frequency distribution of the impulse response. If this distribution exists, then the relation $I(j\omega) = G(j\omega)\cdot V(j\omega)$ between the spectral components of the voltage action and the current reaction have to be correct (among others, *v(t)* has to be absolute integrateable). Effect of the voltage fluctuation - in terms of the system measure – is given by the total energy that had been interchanged between the source of fluctuation and the system reminder. In accordance with the *Parseval* theorem,

$$\begin{aligned} \varepsilon &= \int_{-\infty}^{\infty} v(t)\cdot i(t) dt = \\ &= \frac{1}{\pi}\int_{0}^{\infty} \left|V(j\omega)\cdot I(j\omega)\right| \cdot \cos(\varphi(\omega)) d\omega \end{aligned} . \qquad (2)$$

It should be noted that resulting effect is done only by the apparent power multiplied by the power factor, i.e. real power of individual spectral components. Inverse decomposition of voltage and current to the spectral components yields (3):

$$\varepsilon = \int_{-\infty}^{\infty} v(t)\cdot i(t) dt =$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty}\left(\frac{1}{(2\pi)^2}\int_0^{\infty}\int_0^{\infty} V(j\omega)\cdot I(j\Omega)\cdot e^{j(\omega+\Omega)t} d\omega\cdot d\Omega\right) dt$$

The double integral describes the fact that mutual energies among various spectral components exert its influence by the instantaneous energy interchange between the source of the fluctuation and the loading impedance. However, these deformation energies do not modify the final result.

It can be proved that the source of fluctuation with the constant magnitude of $\left|V\left(j\omega\right)\right| = \sqrt{\dfrac{\pi}{\Delta\omega}}$ in the frequency range $\langle \Omega, \Omega + \Delta\omega \rangle$ would delivered energy 1 Joule to the resistive load 1ohm. Operating to the admittance *G(jω)*, the delivered energy would be

$$\varepsilon(\Omega,\Delta\omega) = \frac{1}{\pi}\int_0^\infty \left|V\left(j\omega\right)\right|^2 \cdot re\left(G\left(j\omega\right)\right)d\omega = \\ = \frac{1}{\Delta\omega}\int_{\Omega}^{\Omega+\Delta\omega} re\left(G\left(j\omega\right)\right)d\omega .$$

For the harmonic excitation at the frequency *Ω*, energy delivered to the load is as follows:

$$\varepsilon_p = \lim_{\Delta\omega \to 0} \varepsilon(\Omega,\Delta\omega) = re\left(G\left(j\Omega\right)\right).$$

The reactive component is interchanged between the source of fluctuation and the load:

$$\varepsilon_q = im\left(G\left(j\Omega\right)\right).$$

Both components can be deducted for arbitrary frequency from the admittance trajectory in the complex plane, see Fig. 2. The stable system keeps steady-state energy exchange with the source of fluctuation. On the frequencies, where $re(G(j\Omega))>0$, the energy passes from the source inside the system. However, such frequency intervals can exist where $re(G(j\Omega))<0$. Then the system gives the energy back to the source. As a limiting case, such frequency can exist where the admittance trajectory touches or intersects negative real axis, i.e. two conditions are true at the same time:

$$re\left(G\left(j\Omega\right)\right) < 0 \wedge im\left(G\left(j\Omega\right)\right) = 0 .$$

In this case, no reactive power is interchanged between the source and the system. The system becomes the supplier of only active power. The cooperation between the source of fluctuation and the rest of system, which ensures steady-state energy exchange, already does not exist. System is on the boundary of stability. In this case, the Fourier transforms of the system variables need not exist. Then also the imittance trajectories *G(jω)* lose their physical meaning.

We can summarize important conclusion for stable systems:

The trajectory of the loading imittance of the source of fluctuation never touches/intersects the negative real axis for stable systems.

Physics of unstable systems

For unstable systems, Fourier transforms of the system responses need not be defined. On the assumption of "exponential-type" responses, we can use their Laplace transforms. For instance, the current reaction $i(t)$ of unstable system to the voltage fluctuation *v(t)* can be (under aforementioned assumption) transformed to the form

$$I(p) = \int_0^\infty i(t)e^{-pt}dt = \int_0^\infty i(t)e^{-\xi t}e^{-j\omega t}dt ,$$

where ξ is some damping factor.

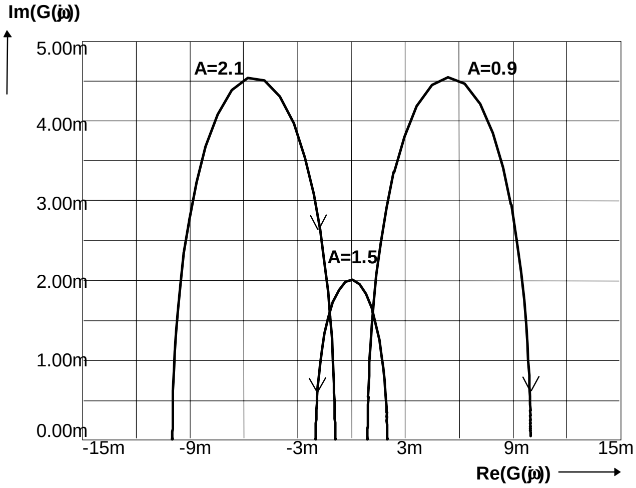

Fig. 2 Trajectory of the output admittance of amplifier on Fig.1.

Fig. 2 shows trajectories of the admittance that represents the load of the fluctuations of the output stage of amplifier on Fig. 1. Trajectories are drawn for values $R_1 = R_2 = 1\text{k}\Omega$ and $C = 1\text{nF}$. For the gain A=0.9, the circuit is stable and the admittance has the negative real pole. The arrows are in the direction of growing frequency from 0 to ∞ Hz. For A=1.5, the circuit is already unstable and the trajectory stops on the negative real axis. For A=2.1, the pole is again negative. However, the trajectory begins on the negative real axis and the circuit seems to be unstable. In reality, this amplifier actually oscillates.

References

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[2] BIOLEK,D.-BIOLEK,Z.: Stability Of Dc Operating Points And Corresponding Dynamical Systems. AMSE, MS'97, Melbourne, pp. 236-240.

Acknowledgement: *This work is supported by the Grant Agency of Czech Republic under Grant No. 102/97/0765.*