DC STABILITY IN CIRCUITS WITH ALGEBRAIC CONFINEMENTS

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Abstract

The DC stability testing of circuits containing macros which implicate algebraic confinements to the nodal voltages, is described in this contribution. The physical background of the stability test is explained which is based on the detection of negative resistances.

1 Introduction

In the course of the design of a new circuit structures or integrated circuits, so-called unstable DC operating points can be a serious problem [1-5]. The DC operating points (in abbr. operating points) are solutions of the algebraic circuit equations after shorting out all inductors and disconnecting all capacitors. The **DC stability is the circuit ability to eliminate *slow* fluctuations round the operating point** which can be caused e.g. by the power supply or temperature fluctuation, gradual modification of the element parameters, etc. After the circuit trajectory is put near a such stable point, it is attracted and the circuit comes to the DC steady state. A general dynamic circuit linearized round a such operating point is said to be **DC stable**. In the case of linearization round an unstable DC operating point, the dynamic circuit is DC unstable [6-15].

Unstable operating points amplify every slow deviation and extrude the trajectory away of the operating point. Such unstable points can be designed either purposely (e.g. flip-flop circuits) or as a consequence of the design mistakes (topology error, neglecting of an essential nonlinearity, etc.) It is not a problem to determine all operating points. However, it is questionable to identify the unstable ones. In the circuit models without reactances the time factor is not presented, which plays an important role in the classical stability theory. Adding a "sufficient amount" of some parasitic reactance elements and observing the stability under the limiting approach when parasitics tend to zero does lead to the required solution. In fact, the DC unstable circuit is unstable for an arbitrary infinitesimal vector of reactance parameters [1]. It is necessary to find the cause of this phenomenon in the thermodynamics. By this reason, the DC circuit can be considered not as a limiting case of the dynamic circuit with sufficiently small reactances, but as a thermodynamic system where the reactances do not exist.

From the thermodynamical point of view, the Ohms law, which is a basic of all DC calculations, is the phenomenological law describing relations between the *average* voltage and the *average* intensity of the charge flowing through a given cross-section. Both of these quantities embody permanent time fluctuations around their average values. It should be noted that in the idealized DC resistive circuit, only the stable equilibrium states with an "attracting ability" can be observable. The "attraction time" or relaxation constant can be well estimated following the thermodynamical system characteristics [16-20].

Since electrical resistance is the important thermodynamical parameter, various methods of finding DC unstable operating points are based on the idea of identification of negative resistances.

2 Method of negative resistances

Consider DC circuit after linearization round investigated operating point. Let us describe it by a conductance matrix $\underline{G}$. Equilibrium state is preserved by a certain power from the supply sources. Fluctuation $\delta\vec{v}$ from the equilibrium state changes the situation in a such way that the average power during the fluctuation will be

$$\delta P = \int_{\Gamma} \delta\vec{i}^{\,T} \cdot d\vec{v} = \int_{\Gamma} \left(\underline{G} \cdot \delta\vec{v}\right)^{T} \cdot \delta\vec{v} \,,$$

where Γ is the trajectory of a deviation. For the DC circuits without any controlled sources, the $\underline{G}$ matric is symmetrical, and

$$\delta P = \frac{1}{2} \cdot \delta\vec{v}^{\,T} \cdot \underline{G} \cdot \delta\vec{v} \,. \tag{1}$$

This equation is well-known as Rayleigh dissipative function, or dissipative potential.

For stable systems, dissipative potential is a positive definitive form, i.e. any fluctuation round the operating point increases the power production. In this way, the stable operating point is settled on the bottom of a "potential hole". On its surface, all casual fluctuations are disposed.

According to the laws of nonequilibrium thermodynamics, the system tends to converge to the state where the energy production is minimum [18,19]. The reaction current plays this role. It operates in the direction of the gradient of dissipative potential

$$\delta\vec{i} = \frac{\partial(\delta P)}{\partial\vec{v}} = \underline{G}\cdot\delta\vec{v}\,. \qquad (2)$$

The operating point corresponds to the *local minimum* of Rayleigh function, if all eigenvalues of the $\underline{G}$ matrix lie on the *right-side complex plane*. For the unstable operating point, the dissipative potential will not have the local minimum here.

As a conclusion of DC stability, the quadratic form (1) has to be positive for *arbitrary* fluctuation. Taking it into account, we can formulate following two equivalent DC stability criteria:

DC operating point is stable if the differential resistance between two arbitrary nodes is positive.
DC operating point is stable if the differential conductance between two points after cutting arbitrary branch is positive.

Method of negative resistance/conductance tests the DC stability by a *single* fluctuation source. We identify the load resistance of the fluctuation source. However, in fact many fluctuations affect the real circuit in the same time. Respecting this fact, we can generalize:

Let many voltage and current fluctuations affect the DC circuit. We join all these fluctuation to the vector $\delta\vec{x}$. The current an voltage circuit reaction measured on the fluctuation sources be joined to the vector $\delta\vec{y}$. Due to circuit linearity

$$\delta\vec{y} = \underline{A}\cdot\delta\vec{x}\,,$$

where $\underline{A}$ is a square matrix. Let us call it ***virtual matrix***. Regarding the fluctuations $\delta\vec{x}$, investigated operating point is DC stable, even if all eigenvalues of the virtual matrix lie on the *right-side* complex plane.

The eigenvalues can be interpreted as the loading imittances of the fluctuation sources. For stable operating points, these imittances have positive real parts.

Testing DC stability by these eigenvalues reliably, we put a set (vector) of the fluctuation sources to such places, from where then can "map" all prospective negative resistances. It is advantageous to choose the number of fluctuations as small as possible, because this is the order of virtual matrix $\underline{A}$.

3 Circuits with the algebraic confinements

Let us consider a macro which is the part of investigated circuit. This macro has *r* outlets; in the DC state, it causes the algebraic confinements regarding the nodal voltages $f(\vec{v}^M)=0$. The superscript ^{M} denotes the nodes where the macro is connected. With respect to the infinitesimal deviations and linearized models, the dependencies can be rewritten to the form

$$f(\vec{v}^M) = f_1 v_1^M + \ldots + f_r v_r^M = \vec{f}^T\cdot\vec{v}^M = 0\,.$$

The circuit reminder out of the macro which is described by the conductance matrix $\underline{G}$ tends to response in the direction of the gradient of dissipative potential. However, now it is bound by the confinement *f*. The circuit functions can be again obtained after finding local extreme of the potential function, but this extreme is now constrained. We apply well-known method of Lagrange multiplicators. Let us solve the extremal problem for the modified Rayleigh function

$$\tilde{R}(\vec{v}) = \frac{1}{2}\cdot\vec{v}^T\cdot\underline{G}\cdot\vec{v} + \lambda\cdot f\,, \qquad (3)$$

where λ is so-called Lagrange uncertain multiplicator. Circuit functions can be derived by differentiation equation (3) with respect to the individual nodal voltages

$$\underline{G}\cdot\vec{v} + \lambda\cdot\frac{\partial f}{\partial\vec{v}} = \underline{G}\cdot\vec{v} + \vec{i}_R = \vec{0}\,, \qquad (4)$$

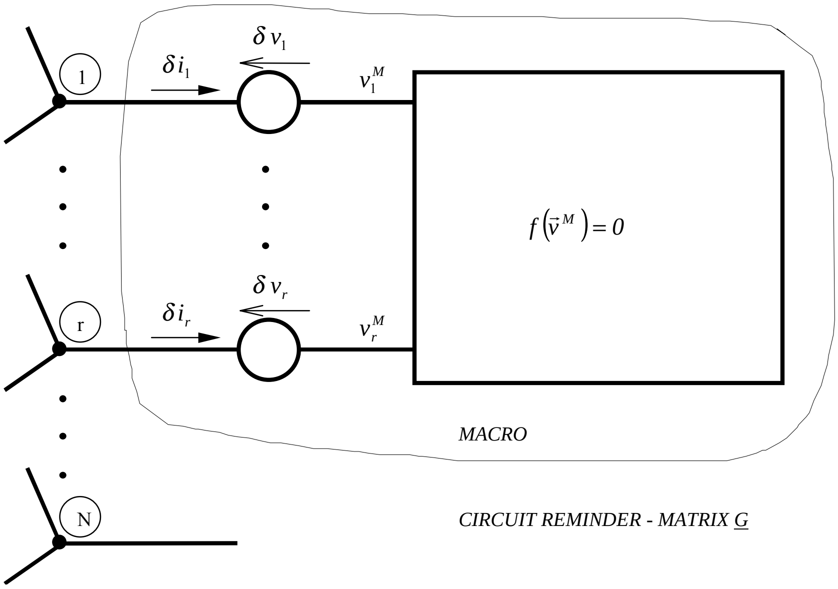

Figure 1: Macro implementation to the circuit.

$\vec{i}_R$ is the current reaction keeping the f condition to be fulfilled. Except of nodal voltages, this reaction is a further unknown quantity. It can be interpreted as a current, which is provided by the macro to the circuit through its outlets. Analysis of equation (4) leads to the conclusion, that macro behaves as a reciprocal element which contributes by all its outlets to the fulfilling of condition *f*. It is due to the fact that we solve extremal problem which has originally arisen in the analytical mechanics. Here the principle of the equality of action and reaction is valid. Nonreciprocal elements like OpAmps try to fulfill given confinement only by some of their outlets, e.g. by the outputs. This fact can be taken into account by the weighed coefficients $0 \le K_i \le 1$, which are assigned to each macro outlets [3]:

$$\underline{G}\cdot\vec{v}+\lambda\cdot\left[f_1K_1 \quad \cdot \quad \cdot \quad f_rK_r \quad \vec{0}^T\right]^T=\vec{0}\,. \tag{5}$$

For the transparency, nodal voltages are numbered in such a way that the first *r* nodes correspond to the macro outlets. That is way the current reaction vector reminder is completed by zeros. Equation (5) along with the confinement condition enables to compute all nodal voltages and the uncertain coefficient λ. Then we can find all currents between the macro and the circuit reminder.

Now we connect source of the voltage fluctuation to each of the macro outlet according to Fig. 1. They are only one outside sources. That is why we will investigate the stability of the zero state. The resulting virtual equations can be compiled as follows:

$$\begin{array}{|c|c|}\hline \underline{G} & \begin{matrix} f_1\cdot K_1\\ \cdot\\ \cdot\\ f_r\cdot K_r\\ \vec{0}\end{matrix} \\ \hline -f_1 \quad \cdot \quad \cdot \quad -f_r \quad \vec{0}^T & 0 \\ \hline \end{array} \cdot \begin{array}{|c|}\hline \vec{v} \\ \hline \lambda \\ \hline \end{array} = \begin{array}{|c|}\hline \vec{0} \\ \hline \delta_u f \\ \hline \end{array} \,.$$

It is true that

$$\delta_u f = f_1\cdot\delta v_1+\ldots+f_r\cdot\delta v_r=\vec{f}^T\cdot\delta\vec{v}\neq 0\,. \tag{6}$$

Considering fluctuations as an inner macro problem, we come to the important conclusion: due to the fluctuations, the macro is not able to fulfill confinement condition properly. Equation (6) offers a microscopic error of the confinement.

Following important equation can be gained from this relation:

$$\lambda = c\cdot\delta_u f\,. \tag{7}$$

Equation (7) gives only the essential dependence between the fluctuation and its consequence. **Reaction to the disequilibrium caused by the fluctuation of an arbitrary value inside the macro is proportional to the rate of the violation of the confinement condition.** Then the constant *c* decides about the stability. For a stable circuit, it has to be positive.

Setting equations for λ and δ*f* yields

$$\delta\ \vec{i}=c\cdot\begin{bmatrix} f_1\cdot K_1\\ \cdot\\ \cdot\\ f_r\cdot K_r\end{bmatrix}\cdot\left[f_1 \quad \cdot \quad \cdot \quad f_r\right]\cdot\delta\vec{v}=\underline{g}\cdot\delta\vec{v}\,\cdot$$

Since the $\underline{g}$ matrix contains only one essential correlation between $\delta\vec{v}$ a $\delta\vec{i}$, it has usually only one nonzero eigenvalue. For DC stability, this number has to lie on the right-side complex plane.

In general, macros can put more algebraic confinements to the circuit variables. Each constrains means one Lagrange multiplicator. Number of the eigenvalues (=number of the imittances which decide about the DC stability) will be equal to the number of constrains. For example, OpAmp sets out one imittance to the circuit, two-port two imittances.

4 Conclusion

A new DC stability test is described. This test is based on the finding of constrained extreme of the circuit dissipative potential after circuit completing by macros. Included macro means the additional algebraic constrains in the DC equilibrium state. By means of the weighted coefficients assigned to the macro outlets we can model various macro properties as finite OpAmp input resistance etc. The procedure how to find weighted coefficients which are responsible for the stability (equation (7)) can be easy algorithmized.

Method utilizing the Lagrange multiplicators has very interesting physical interpretation, which will be discussed in an individual contribution in more details.

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