STABILITY OF DC OPERATING POINTS AND CORRESPONDING DYNAMICAL SYSTEMS

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Abstract - Classical methods of the stability testing may fail for the circuits with controlled sources. They are also inconvenient for the stability testing of DC operating points. Solution using the thermodynamics approach is proposed in this contribution. As a result, the unified stability criterion for both resistive and dynamical systems is formulated.

Key words - operating point, potential stability, virtual eigenvalues, dynamical circuit.

INTRODUCTION

It is shown in [1] that the stability testing of linearized circuits with the controlled sources using well-known criteria based on the testing of poles location can lead to the results that do not correspond to the reality. In some cases, all poles can lie on the left half-plane in spite of the evident circuit instability [2]. Green showed in [1] that the cause can consist in the wrong modeling of the inertia of the controlled sources and the absence of prospective parasitic reactance elements that play important role in the stability of investigated circuit. For this reason, he divided whole problem to two parts: We investigate so-called potential stability of dc operating point of resistive circuit and the stability of corresponding dynamical circuit after augmenting of resistive circuit by reactance elements. Dc operating point can be either potentially stable or unstable. Unstable operating point cannot be stabilized by augmenting resistive circuit by the reactance elements. The potential stability is the necessary but not sufficient condition of the stability of corresponding dynamical circuit.

In [1], Green described so-called „Γ-test“ as an algorithm of identifying of unstable DC operating points. However, this algorithm is based on the necessary bur not sufficient condition of instability. As a result, some of unstable operating points cannot be identified. Therefore we published in [3] the theory of so-called virtual dynamics of resistive circuits which leads to the general criterion of potential stability of DC operating point. We summarize main results.

First, we select circuit variables virtual shiftings of that will deflect the circuit from *DC* operating point. Second, we model virtual shiftings of these variables. Third, we observe circuit reaction - evolution of the virtual shiftings as an indefinite chain of causes and consequences. This evolution passes in so-called space of virtual shiftings (or virtual space). Let us call it virtual trajectory. For the potentially stable operating point, virtual trajectories have to converge to the origin of virtual space.

Likewise as in the classical trajectories of dynamic circuits, the character of virtual trajectories can be evaluated by so-called (virtual) eigenvalues. Stability testing of *DC* operating point can be performed using the location of virtual eigenvalues in the complex plane.

In our contribution, we extend the method of the virtual eigenvalues to the dynamical circuits. Resulting method is the efficient tool for the stability testing of circuits with controlled sources and it includes the testing of DC operating points as boundary case. The relations between the potential stability of dc operating point and the stability of corresponding dynamical circuits will be explained.

THERMODYNAMICAL STARTING POINTS

During the motion, the isolated system is controlled by the principle that has been recently formulated by *Prigogine*:

During the evolution, the system tends to the state with the minimum entropy production.

After system being deflected from its stable DC operating point, such motion is done that the entropy production will fall in time. The minimum production of the entropy will be reached at the operating point. Thus, the entropy production represents scalar potential which characterizes system evolution round the operating point.

The DC operating point can be defined by the set of selected voltages and currents. From the thermodynamical point of view, they are only macroscopic parameters (average values) describing statistical properties of the flow of large amount of the current carriers. **Fluctuation** is the deviation of the microscopic parameter from its average value (i.e. from the macroscopic parameter). Following statement results from the principle of minimum entropy production:

Arbitrary fluctuation that deflects the system from the stable equilibrium state always increases the system energy.

This law can be considered as the general starting point to define stability of the equilibrium state and the DC operating point.

STABILITY OF THE DC RESISTIVE CIRCUITS

Let us consider electric circuit where the reactances are negligible. The circuit is in one of possible dc operating points. Applying statement mentioned above yields the definition of stability:

Definition 1:

DC operating point of resistive circuit is stable if the circuit is consumer for arbitrary fluctuation.

This definition yields following method of stability testing:

Method 1:

We include voltage source to the arbitrary branch or current source between two arbitrary nodes. Then the operating point is stable if power of these sources is always positive.

It should be noted that each dc (linearized) circuit, which consists only of positive resistors is always stable. These conclusion results from the positive definity of circuit matrices regardless of the analysis method (modified nodal analysis, method of loop currents etc.). There are two possible causes of instability: resistors with negative resistance and controlled sources.

Described method is not suitable for the stability testing of large circuits. Following consideration shows that it is not necessary to include sources of fluctuation to each branch and between each two nodes. The principle of instability is shown in Fig. 1 on the example of voltage controlled current source (VCCS). The feedback loop between the controlled source and its controlling input is decisive for the dc operating point stability. It is sufficient for the stability test if the fluctuation will affect this loop. From both fundamental and practical reasons, it is convenient to consider directly the fluctuation of controlled source (i.e. its "power" part).

The practical method of stability testing can be based on the idea mentioned above. Let us assume that the resistive nonlinear circuit is linearized round investigated operating point. As a result, linear resistive circuit with controlled sources is obtained. Negative resistances will not be assumed in following considerations, because they can be modeled by the controlled sources.

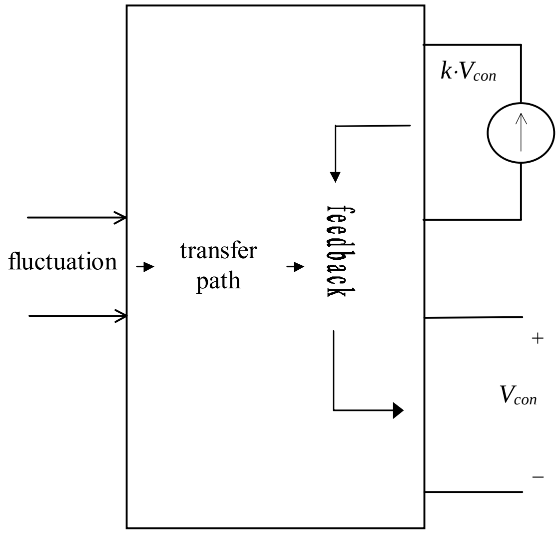

Fig. 1. To the problem how to choose location of the source of fluctuation.

Method 2:

- *We add voltage source of fluctuation δV_i in series to each controlled voltage source V_i.*
- *We add current source of fluctuation δI_i in parallel with each controlled current source I_i.*
- *We arrange voltage and current fluctuations to the vector δX:*

$$\delta \mathbf{X} = [\delta V_1 \ \delta V_2 \ldots \delta I_1 \ \delta I_2 \ldots]^T = [\delta \mathbf{V} \ \delta \mathbf{I}]^T$$

- *We compute circuit reactions to the fluctuations: currents δI_{Ri} of voltage fluctuation sources and voltages δV_{Rj} of current fluctuation sources (we respect the source orientation of voltages and currents). Due to linearity, reaction is proportional to action:*

$$\delta \mathbf{X}_R = \mathbf{H} \delta \mathbf{X} \tag{1}$$

where

$$\delta \mathbf{X}_R = [\delta I_{R1} \ \delta I_{R2} \ldots \delta V_{R1} \ \delta V_{R2} \ldots]^T = [\delta \mathbf{I}_R \ \delta \mathbf{V}_R]^T$$

is vector of reactions and ***H*** *is so-called fluctuation matrix. This matrix can be compiled algorithmically from the circuit topology.*

- *We apply* **stability criterion:**

Operating point is stable if all eigenvalues λ of the fluctuation matrix H lie on the right complex half-plane, i.e. if

$$\forall i = 1, \ldots N : \mathrm{Re}\{\lambda_i\} > 0 \tag{2}$$

where N is number of controlled sources.

Proof:

According to *Definition 1*, total power δP consumed by the resistive circuit from the sources of fluctuation must be positive:

$$\delta P = \sum_i \delta V_i \cdot \delta I_{Ri} + \sum_j \delta I_j \cdot \delta V_{Rj} > 0 \cdot \qquad (3)$$

or using the vector form and equation (1)

$$\delta P = \delta \mathbf{X}^T \mathbf{H} \delta \mathbf{X} > 0 \qquad (4)$$

The total power is the quadratic form (4) of the vector of fluctuations. To be positive for all nonzero fluctuations, the eigenvalues of matrix **H** have to fulfill condition (2). ■

EXAMPLE 1

The stability of circuit in Fig. 2 (a) is investigated in [1]. The linearized circuit including the additional sources of fluctuation is in Fig. 2 (b). As a result of [1], operating point is stable. We compare it with our analysis.

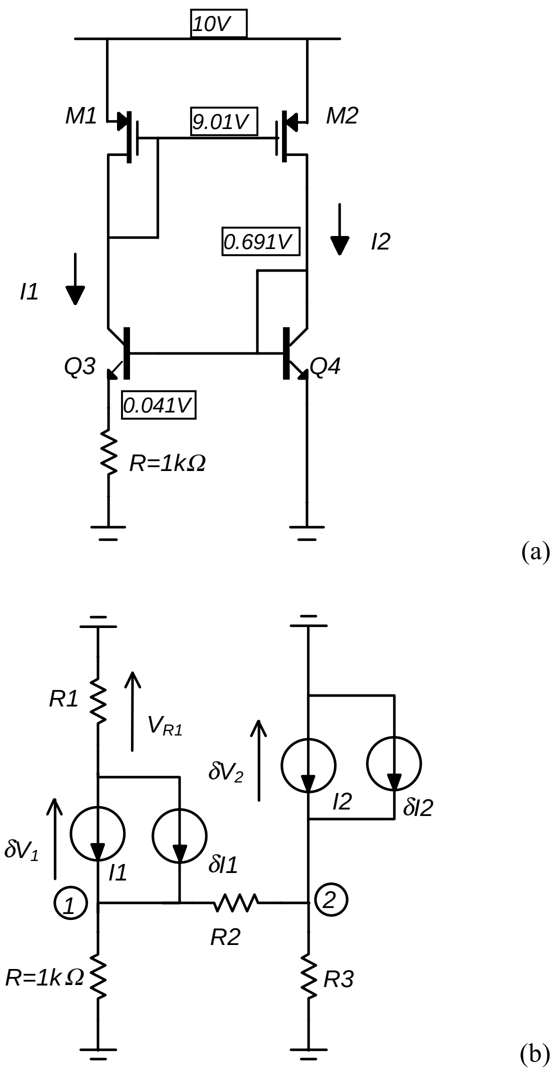

Fig. 2. (a) Nonlinear circuit and the coordinates of its operating point, (b) linearized model including the sources of fluctuation.

For given operating point, following parameters are true:

R_1=3.51kΩ, R_2=64kΩ, R_3=641Ω, $I_1=k_1(V_2-V_1)$, $I_2=k_2V_{R1}$, constants k_1=1.57mS and k_2=285μS.

Variations of current sources create the vector of "action" δ**X**, variations of voltages across these sources create the vector of "reaction" δ$\mathbf{X}_R$:

$$\delta \mathbf{X} = [\delta I_1 \; \delta I_2]^T, \; \delta \mathbf{X}_R = [\delta V_1 \; \delta V_2]^T$$

Analyzing circuit in Fig. 2 (b) yields

$$\begin{bmatrix} \delta I_1 \\ \delta I_2 \end{bmatrix} \doteq \begin{bmatrix} 5{,}66421.10^{-4} & 1{,}55456.10^{-3} \\ -2{,}19143.10^{-4} & 1{,}57208.10^{-3} \end{bmatrix} \cdot \begin{bmatrix} \delta V_1 \\ \delta V_2 \end{bmatrix} .$$

Comparing this result with equation (1), the square matrix on the right side is the inversion of fluctuation matrix **H**. It is known that if all eigenvalues of sqare matrix lie on the right-side complex plane, then after inversion resulted matrix has the same property. For this reason, we need not do inversion of matrix in the equation mentioned above before computation of eigenvalues and application of the stability criterion. The right-side matrix has two eigenvalues:

$$\lambda_{1,2} \approx (10.69 \pm j \cdot 2.964) \cdot 10^{-4} \text{ [S]},$$

i.e. dc operating point of tested circuit is stable. This conclusion is in accordance with [1].

We can even visualize the circuit motion after deflection from dc operating point due to the fluctuation. So called virtual trajectories corresponded to investigated circuit are in Fig. 3. We can see that operating point is stable because the virtual trajectories have the character of stable focus. For more details concerning the construction of virtual trajectories see [4].

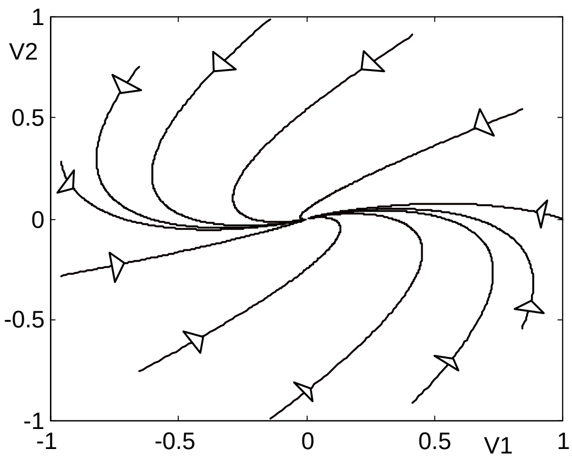

Fig. 3. Virtual trajectories of circuit in Fig. 2(a). Stable operating point has coordinates [0 0].

STABILITY OF THE DYNAMICAL CIRCUITS

In case of real circuits, the spreading of fluctuations is filtered and modified by reactance elements. For stability testing, it is suitable to consider that:

- fluctuations are everywhere,
- in general, fluctuations can have unlimited frequency spectrum $\delta\mathbf{X}(j\omega)$.

Obviously, the power consumed/generated by circuit will now depend on the frequency of spectral components of the fluctuation.

Definition 2:

Circuit is unstable if such spectral component of fluctuation exists for which the circuit behaves as a source of only active power.

In other words, for stable circuit, such frequency does not exist when the pure active power is transferred from the circuit to the source of fluctuation.

Proof:

Is beyond the scope of this paper.

The fluctuation matrix **H** and its eigenvalues will now depend on frequency. The eigenvalues can be now drawn in the complex plane as a trajectories wit the parameter ω. Then following statement is true:

Stability criterion:

Circuit is stable if and only if the trajectories of all eigenvalues $\lambda_i(j\omega)$ of the fluctuation matrix H do not touch negative real half-axis of the complex plane, i.e.

$$\forall i \in \langle 1,...,N\rangle \text{ and } \forall \omega \in \langle 0,\infty\rangle : \quad \text{if } \operatorname{Im}\{\lambda_i(j\omega)\} = 0 \text{ then } \operatorname{Re}\{\lambda_i(j\omega)\} > 0 . \tag{5}$$

It should be noted that for $\omega = 0$ we obtain the criterion (2) for the stability of dc operating point.

This criterion has been published in [5] in the simplified (and not general) form as so called “lambda-criterion”.

EXAMPLE 2

Consider simple circuit with VCVS in Fig. 4 (a), with parameters C1 =C2 =1nF, R1 = 100Ω, R2 = 1kΩ. First assume ideal controlled source with constant gain *A* for all frequencies. For A = 12, the real behavior of this circuit is unstable due to positive feedback. However, the pole of corresponding circuit is negative:

$$s_p = -\frac{1/R_1 + 1/R_2(1-A)}{C_1 + C_2(1-A)} = -10^5 s^{-1}$$

The solution of this „paradox“ consists in the fact that the DC operating point is unstable. In conformity with our theorems, the real behavior of investigated circuit is then unstable. The criterion based on the pole location fails.

Now let us use our approach. Output of controlled sources will be completed by virtual voltage source as shown in Fig. 4 (b). For given frequency, the circuit has single eigenvalue λ_R:

$$\delta V(j\omega) = \lambda_R(j\omega)\delta I(j\omega)$$
$$\lambda_R(j\omega) = \frac{R_1}{1 + j\omega R_1 C_1}(1-A) + \frac{R_2}{1 + j\omega R_2 C_2} \tag{6}$$

For positive frequencies, the eigenvalue mapping to the complex plane is in Fig. 5 (a). We can see that investigated circuit is unstable because there are two frequencies where the trajectory touches negative real half-axis. DC operating point is also unstable because $\lambda_R(\omega = 0) < 0$.

Considering low-pass character of VCVS with the cutoff frequency 1 MHz, the method of pole location will give correct results. There will be two poles

$$s_{p1} \doteq -1{,}21.10^5 s^{-1}, s_{p2} \doteq 2{,}6.10^7 s^{-1}$$

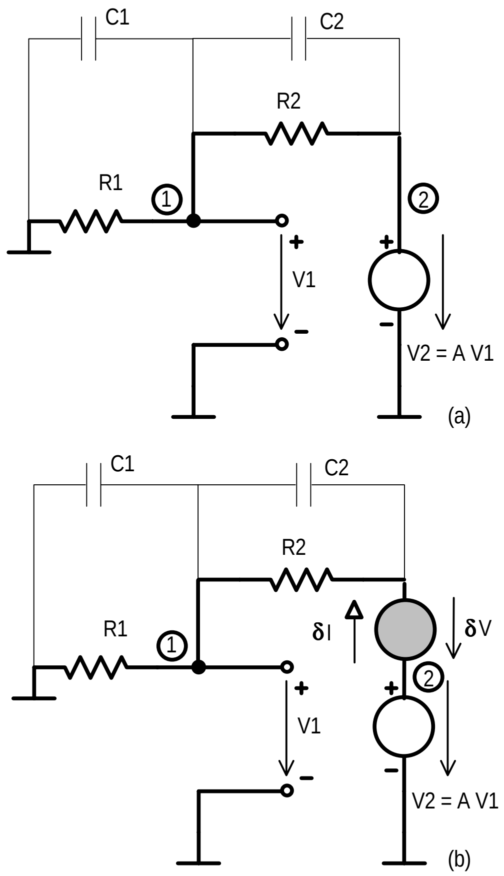

Fig. 4. (a) investigated circuit, (b) consideration of the source of fluctuation.

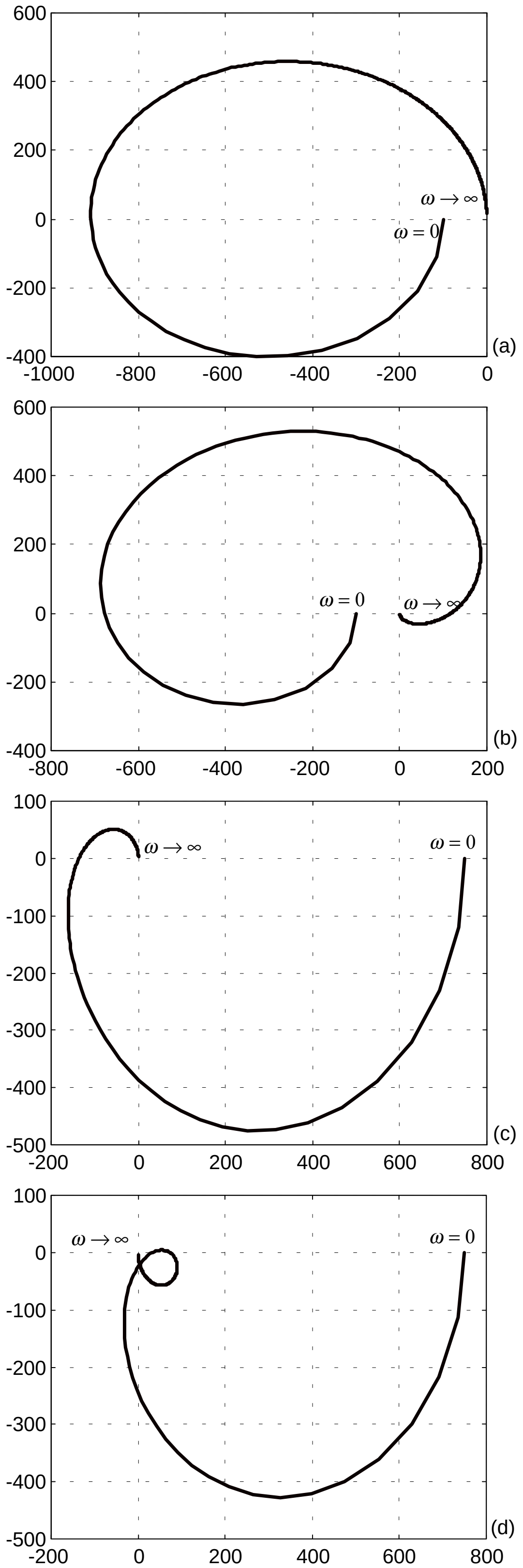

Fig. 5. Trajectories of virtual eigenvalue for the circuit in Fig. 4. (a) $A = 12$ allpass (unstable), (b) $A = 12$, $f_C = 1$ MHz (unstable), (c) $A = 3{,}5$ allpass (unstable), (d) $A = 3{,}5$, $f_C = 1$ MHz (stable).

The positive pole indicates that circuit is unstable. This fact is verified by the trajectory in Fig. 5 (b).

For small but allpass gain $A = 3{,}5$, the circuit has single positive pole

$$s_p = 5.10^6 s^{-1}$$

Corresponding trajectory in Fig. 5 (c) indicates, that:

- DC operating point is stable, but
- in the hypothetical case of allpass gain, the dynamical circuit would be unstable.

Considering the cutoff frequency 1 MHz yields two poles

$$s_{p1,2} \doteq (-0{,}3938 \pm j4{,}8381).10^6 s^{-1}$$

As shown in Fig. 5 (d), the resulting dynamical circuit would be now stable.

Conclusion

The main ideas of the fluctuation theory and its application to the stability investigation is described. This approach is based on the fact that the equilibrium point of arbitrary system is permanently disturbed by diverse fluctuations. The stable operating point is the result of internal control processes that stabilize fluctuations. Investigated system is unstable if such spectral component of fluctuation exists for which the system behaves as the source of only active power.

Given method offers right results in the areas where the classical “pole-location” methods can fail (circuits with the controlled sources), and where no general tools are known (stability of dc operating points of resistive circuits). Described approach provides unified view to the stability of dc operating point and the stability of corresponding dynamical circuits.

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