Virtual Dynamics of DC Circuits

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Abstract **-** It is shown in this contribution that besides the classical dynamics caused by reactance elements, each *DC* resistive circuit has its special hidden so-called virtual dynamics which contains information about the *DC* operating point stability. Virtual eigenvalues can be attached to each linear model of resistive circuit linearized around its *DC* operating point. Corresponding virtual trajectories describe motion of operating point in the virtual space beeing deflected from the equilibrium state and they help to decide whether the operating point is stable.

INTRODUCTION

In [1] and [2], the definition of so-called potential stability of *DC* operating point is introduced. The main ideas of this definition are as follows:

1. The nonlinear circuit is linearized around an investigated operating point. The linear resistive circuit with controlled sources is obtained.
2. A *DC* circuit's operating point is said to be potentially stable if, by inserting some set of positive-valued shunt capacitors and series inductors into the linear resistive circuit, the corresponding equilibrium point of the resulting dynamic circuit is stable, and if the equilibrium point of any dynamic circuit created by augmenting this dynamic circuit with an arbitrary set of shunt capacitors and series inductors, whose values are sufficiently small, is also stable.
3. An operating point that is not potentially stable is said to be unstable.

This definition of potential stability has been generalized in [3].

On the basis of this concept of potential stability, the algorithm to identify unstable operating points is proposed. The algorithm uses only algebraic equations of resistive linear circuit and completes single number Γ. If $\Gamma < 0$, the corresponding operating point is unstable. If $\Gamma > 0$, we are not able to decide about the stability using this method.

Let us call method mentioned above as „Γ - test“. It can be shown that Γ - test is based on the sufficient but not necessary condition of operating point instability. As a result, not all unstable operating points can be identified.

The essential question appears whether the test of the *DC* operating point stability can be done without considering reactance elements in the investigated circuit. It is shown in [1] and [3] that:

- Stable/unstable dynamic linear circuit with controlled sources can be done unstable/stable by inserting some set of positive-valued shunt capacitors and series inductors, but
- If circuit contains so-called inertia-type controlled sources only [3], then if operating point is unstable, the corresponding dynamic circuit is also unstable, regardless of whether or not any additional outside capacitors and inductors are modeled.

The inertia-type controlled sources mean ones with natural real inertia which is modeled

- by capacitors in parallel with each dependent current source and with each input branch of voltage-controlled source and
- by inductors in series with each dependent voltage source and with each input branch of current-controlled source.

In other words, the Γ - test and the whole conception of potential stability are based on the assumption of inertia-type controlled sources which is naturally fulfilled in real circuits.

In [4,6-7], the necessary and sufficient condition of *DC* operating point instability is formulated using so-called virtual eigenvalues of resistive circuits with controlled sources.

First, we summarize main ideas of virtual eigenvalues. Second, we formulate stability theorem. Third, we present illustration example how to investigate stability of *DC* operating point using the virtual eigenvalues and virtual trajectories.

VIRTUAL DYNAMICS OF DC RESISTIVE CIRCUITS

DC steady state is the result of forces affecting *DC* resistive circuit that are in a balance. What happens if the balance is violated by the small deflection of circuit from its equilibrium state? In case of potentially stable operating point, internal mechanism causes the circuit motion back to the original point. For unstable operating point, the circuit will recede from this point. The main problem is that this motion cannot be investigated in time evolution due to absence of differential equations of *DC* resistive circuit. In spite of that we can visualize the circuit motion as a result of internal forces acting after the circuit having been deflected from its previous operating point.

Let us consider linear resistive circuit with *N* controlled sources as a result of linearization of nonlinear circuit around investigated operating point. In reality, these sources control energy exchange between the circuit and outside sources. For stable *DC* operating point, the *DC* steady-state means the dynamic equilibrium when the circuit responds to the permanent disturbances of internal voltages and currents. These disturbances can be modeled using so-called virtual shiftings (or variations) [5]. From the thermodynamics, virtual shifting is some mathematical probe which enables to realize following thought experiment: what would be happen if certain circuit variable would change its steady value *V* by the variation δV? If the circuit will affect against (toward) this caused variation, the operating point is potentially stable (unstable).

In conformity with the original thermodynamical concept, the virtual shifting need not pass in time and is more general than the time variation of circuit variable. In this way, we can avoid problematic time factor which is necessary for the classical stability testing of dynamic circuits.

First, we select circuit variables virtual shiftings of that will deflect the circuit from *DC* operating point. In conformity with the idea that dependent sources control energy exchange between the circuit and outside sources, these circuit variables will be outputs of controlled sources. Second, we model virtual shiftings of these variables. Third, we observe circuit reaction - evolution of the virtual shiftings as an indefinite chain of causes and consequences. This evolution passes in so-called space of virtual shiftings (or virtual space). Let us call it virtual trajectory. For the potentially stable operating point, virtual trajectories have to converge to the origin of virtual space.

Likewise as in the classical trajectories of dynamic circuits, the character of virtual trajectories can be evaluated by so-called (virtual) eigenvalues. We show that stability testing of *DC* operating point can be performed using the location of virtual eigenvalues in the complex plane.

STABILITY THEOREM

Let us complete each controlled voltage (current) source by virtual voltage (current) source δV_A (δI_A) as illustrated in Fig. 1. The subscript $_A$ means „action". The circuit reaction is in the form of virtual current δI_R (virtual voltage δV_R).

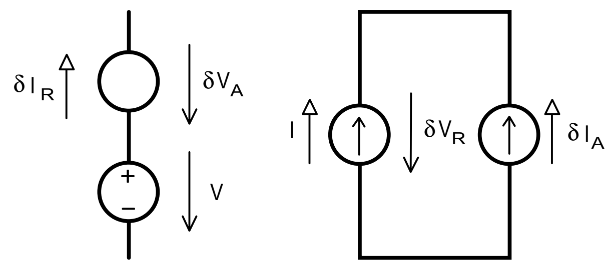

Fig. 1. Virtual voltage (current) sources δV_A (δI_A) and circuit virtual reaction δI_R (δV_R).

All virtual voltages (currents) create the vector of virtual voltages $\delta\mathbf{V} = [\delta\mathbf{V}_A \quad \delta\mathbf{V}_R]^T$ (vector of virtual currents $\delta\mathbf{I} = [\delta\mathbf{I}_A \quad \delta\mathbf{I}_R]^T$). So-called *equations of virtual shiftings* can be compiled:

$$\delta\,\mathbf{V} = \mathbf{R}\delta\,\mathbf{I} \;\text{ or }\; \delta\,\mathbf{I} = \mathbf{G}\delta\,\mathbf{V} \qquad (1)$$

where **R** (**G**) is the *N*x*N* regular matrix of virtual resistances (conductances). *N* is the number of controlled sources. Let us call eigenvalues λ_R (λ_G) of matrix **R** (**G**) as resistance (conductance) virtual eigenvalues. We formulate

necessary and sufficient condition of the potential stability of the DC operating point:

All virtual eigenvalues λ_R (or λ_G) have to lie on the complex right half-plane.

Proof of this theorem is beyond the scope of this contribution. For circuits with voltage-controlled voltage sources, the proof is in [7].

EXAMPLE

Let us try to decide whether the operating point corresponding to the linearized circuit in Fig.2 (a) is potentially stable or not for ($A_1 = 2$, $A_2 = 2$) and ($A_1 = -2$, $A_2 = 2$).

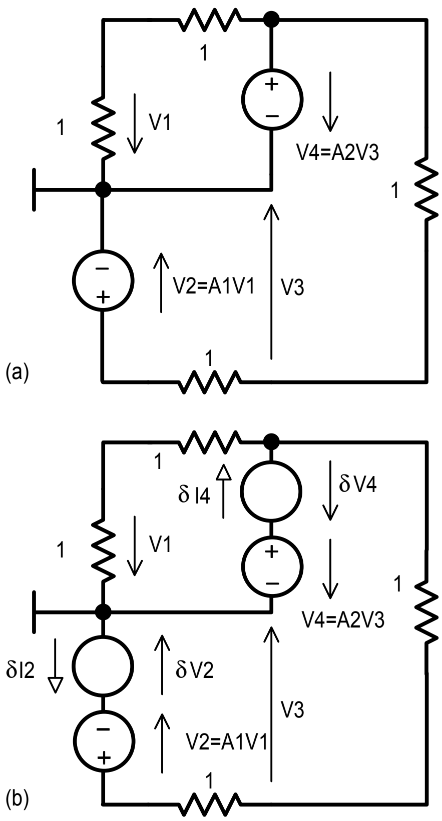

Fig. 2. (a) circuit under test, (b) circuit completed by virtual sources.

The Γ - test gives sufficient condition of the operating point instability:

$$\Gamma = 1 - 0.5A_2 - 0.25A_1A_2 < 0 \tag{2}$$

For $A_1 = 2$, $A_2 = 2$, $\Gamma = -1 < 0$ and the operating point is unstable.

For $A_1 = -2$, $A_2 = 2$, $\Gamma = 1 > 0$ and we cannot decide about the stability.

Now the stability test will be performed using the method of virtual eigenvalues. First, the circuit is completed by virtual sources - see Fig. 2(b). Resistance equations of virtual shiftings are

$$\underbrace{\begin{bmatrix} \delta V_2 \\ \delta V_4 \end{bmatrix}}_{\delta \mathbf{V}_A} = \underbrace{\begin{bmatrix} 4 - A_1 & 2 - A_1 \\ 2 - 3A_2 & 2(1 - A_2) \end{bmatrix}}_{\mathbf{R}} \underbrace{\begin{bmatrix} \delta I_2 \\ \delta I_4 \end{bmatrix}}_{\delta \mathbf{I}_R} \tag{3}$$

For $A_1 = 2$, $A_2 = 2$, eigenvalues of matrix **R** are

$$\lambda_{R1} = 2, \quad \lambda_{R2} = -2$$

Because of one negative eigenvalue, the operating point is unstable. This statement is in conformity with the Γ - test.

For $A_1 = -2$, $A_2 = 2$, both virtual eigenvalues are positive:

$$\lambda_{R1} = \lambda_{R2} = 2$$

and we can conclude that the operating point is potentially stable. Note that the Γ - test has failed in this case.

Now let us compare the Γ - test to the method of virtual eigenvalues from the point of view of following question: How to choose amplifications A_1 and A_2 to ensure (un)stable operating point?

For every combination of amplifications [A_1, A_2], the operating point is either potentially stable or unstable. In coordinates [A_1, A_2] we can select regions where the given method can identify the unstable operating point. For the Γ - test, this region is described by equation (2). We can see it on Fig. 3(a) as a shaded area. However, some unstable operating points lie outside this area and cannot be identified.

Similar area can be drawn for the test using virtual eigenvalues. Computing eigenvalues of the matrix **R** from equation (3) and taking into account that at least one of them has to lie on the complex left half-plain leads to necessary and sufficient conditions of the operating point instability:

$$4 - 2A_2 - A_1A_2 < 0 \text{ and } A_1 + 2A_2 - 6 > 0 \tag{4}$$

The first inequality is equivalent to the condition (2) of Γ - test. The second inequality expands the area in which unstable operating points can be identified as compared with the Γ - test. We can be sure that the shaded area in Fig. 3(b) corresponds to all unstable operating points because it is based on necessary and sufficient conditions (4) of instability.

In Fig. 4, virtual trajectories corresponding to som combinations A_1 and A_2 are drawn. Virtual trajectories have been obtained by the numerical solution of the equation

$$\delta \mathbf{V}_{A,k+1} = \delta \mathbf{V}_{A,k} - r\,\delta \mathbf{I}_{R,k} = \left(\mathbf{E} - r\,\mathbf{R}^{-1}\right)\delta \mathbf{V}_{A,k}$$

where **E** is the unity matrix and r is a positive small constant (for more details, see [7]).

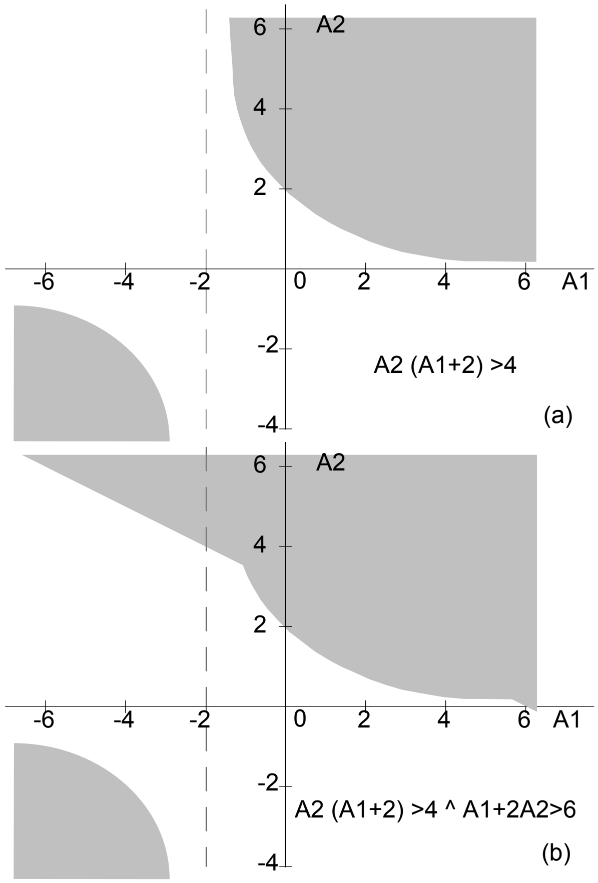

Fig.3. Regions where unstable operating points can be identified using the (a) Γ - test, (b) method of virtual eigenvalues.

ACKNOWLEDGEMENT

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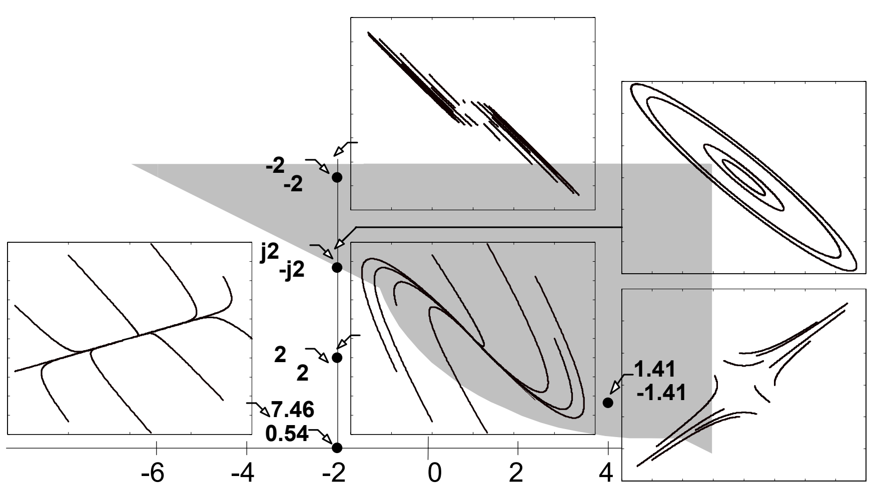

Fig. 4. Virtual trajectories for given eigenvalues.