STABILITY, RECIPROCITY AND CONTROLLABILITY OF DC CIRCUTS

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Abstract

The relationship between the DC instability of electrical circuits [1] and the nonreciprocal couplings acting inside these circuits is analyzed. Such couplings can often be accomplished by controlled sources. The DC circuit is defined as a thermodynamical system which responds to fluctuations round the equilibrium point in the direction of so-called virtual trajectory. The influence of nonreciprocal couplings on the shape of these trajectories is investigated. As a result, we prove a direct connection between the DC instability and the system noncontrollability with regard to the equilibrium state.

Basic terms of DC stability

The DC stability is a circuit ability to cope with infinitely *slow* disturbances pelting its equilibrium state. The DC unstable points are so-called hidden equilibrium points, which are normally unmaintainable by virtue of DC “positive feedback”, well-known for instance in comparators or flip-flop circuits. Such “hidden” states frequently occur in most real circuits containing semiconductor devices. They affect the circuit’s total dynamics, transients to the operating points, etc. An arbitrary *slow* deflection from the unstable equilibrium leads to the amplification of this deflection, and the circuit trajectory is forced out of the equilibrium point. Since the mechanism of instability is realized via DC components of circuit variables, reactance elements do not partake in it, i.e. they behave as inactive. The DC equilibrium states are determined by solving algebraic equations of DC circuit with disconnected capacitors and shunted inductors.
The following propositions are true for relating the DC and general (in)stability [1]:

Proposition 1: The DC-stable circuit can become unstable by being completed with reactance elements.
Proposition 2: The DC-unstable circuit cannot become stable by being completed with reactance elements to the minimum-phase circuit.

The first proposition reminds us that a circuit can often oscillate due to parasitic capacitances and/or inductances. The second proposition is of practical importance because identifying the DC unstable point means nearly always detecting generally an unstable point. However, identifying DC unstable points is a well-known problem. The problem is in that the DC circuit is described by algebraic and not differential equations, and the time factor is not present here. Nevertheless, the information on DC instability is „encoded“ in the topology and in DC parameters of the circuit, and it is manifested by the effect of “negative resistance”.

Virtual trajectory

Considering DC circuit as a thermodynamic system brings a new view into the problem of DC stability. This conception enables us to study *trends* instead of real trajectories. Let us start from the principle of minimum entropy production. For DC circuits it can be formulated as follows:

Proposition 3: The DC circuit tends to move against the direction of the gradient of dissipative potential.

The dissipative potential is a scalar function which shows how the circuit copes with permanent fluctuations round the equilibrium point. The DC stable/unstable point is then located in the local minimum (maximum) of dissipative potential. The trajectory "seeks" such a continuation that would minimize dissipative potential in the "following step".
Starting from the modified nodal analysis, this potential round the equilibrium state can be expressed for reciprocal circuit in a quadratic form which is given by conductance matrix $\underline{G}$:

$$\Phi(\vec{u}) = \frac{1}{2}\vec{u}^T \cdot \underline{G} \cdot \vec{u} - \vec{u}^T \cdot \vec{i} \ ,$$

where $\vec{i}$ is the outside excitation. The motion equations are obtained by solving the extremal problem

$$\nabla\Phi\big|_{\delta \vec{u}} = \vec{0} \ . \tag{1}$$

According to Proposition 3, the trajectory of an autonomous system is directed from the point of deflection $\delta\,\vec{u}$ to the point $\delta\,\vec{u} - d\,\vec{u}$, where

$$d\,\vec{u} \approx \nabla\Phi\big|_{\delta \vec{u}} = \underline{G} \cdot \delta\,\vec{u} \ . \tag{2}$$

A set of so-called virtual trajectories could be constructed along the direction field (2). On these trajectories, a pure DC system would either dissolve or evolve fluctuations round the equilibrium state. The character of these trajectories will be given unambiguously by the eigenvalues of matrix $\underline{G}$ according to the following proposition:

Proposition 4: Virtual trajectories will be terminated in the equilibrium point just when all the eigenvalues of matrix $\underline{G}$ lie in the right-half complex plane.

A consequence is the condition of DC stability

$$\delta\,\vec{u}^T \cdot \underline{G} \cdot \delta\,\vec{u} > 0$$

which must be true for an *arbitrary* fluctuation $\delta\,\vec{u}$.
According to equation (2), dissolving or evolving the fluctuation is ensured by currents between the voltage nodes and the ground. In reality, these currents can flow through a parasitic capacitance C_i between node $\underline{i}$ and the ground. In matrix representation, $\underline{C}=diag[C_i]$. Then the real trajectory is given by solving equation

$$\underline{C} \cdot \dot{\vec{u}} + \underline{G} \cdot \vec{u} = \vec{0} \ .$$

As proved easily, if all the parasitic capacitances were identical, the real system trajectory would correspond to the virtual trajectory. The real trajectory is thus determined by the thermodynamic features of DC circuit. The reactance elements modify these trajectories additionally and they can even lead to instability of the given state.

DC circuit with algebraic couplings

Many circuit elements introduce algebraic couplings of the type of $f(\vec{u}) = 0$ into state variables. For example, the ideal voltage amplifier couples its input and output voltages as follows:

$$f = U_2 - A \cdot U_1 = 0 \ . \tag{3}$$

Fig. 1 demonstrates a number of methods how to bring about this condition. It is substantial where we place the executive element (VCVS in our case) which satisfies condition (3). The cases (a) and (b) are trivial, in (c) the condition is satisfied by two controlled sources, where each of them can participate by a different weight.
From an other point of view, fulfilling condition $f = 0$ is accomplished by general macros only by (a) a current to node ① , (b) a current to node ②, (c) currents to both nodes ① and ②.

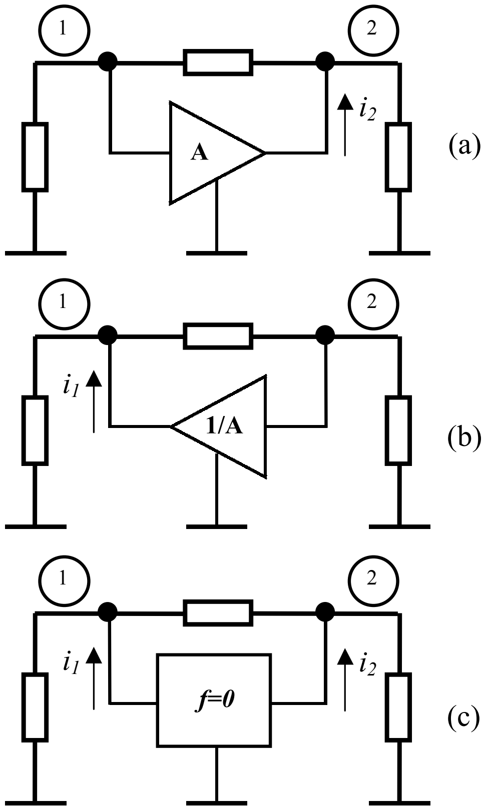

Fig. 1. Coupling realizations.

The synthesis of general solution (c) results from solving the extremal problem (1) while retaining condition (3). We find such a virtual trajectory that will minimize dissipative potential (the surface in Fig. 2) and which, at the same time, will remain on the hyperplane $f(\vec{u})=0$ (the coupling line $U_2 - A \cdot U_1 = 0$ in Fig.3). This task of constrained extreme can be solved by so-called Lagrange multipliers λ. The motion equations are derived from the gradient of modified potential

$$\nabla\Phi^*\Big|_{\delta\,\vec{u}} = \nabla(\Phi - \lambda f)\Big|_{\delta\,\vec{u}} = \vec{0}$$

otherwise

$$\underline{G} \cdot \vec{u} - \lambda \frac{\partial f}{\partial \vec{u}} = \vec{0} .$$

Performing the differentiations yields $i_1 = -A \cdot \lambda$, $i_2 = \lambda$. The indeterminate multiplier represents a current supplied by a source, which serves as a reference for driving currents i_1 and i_2.

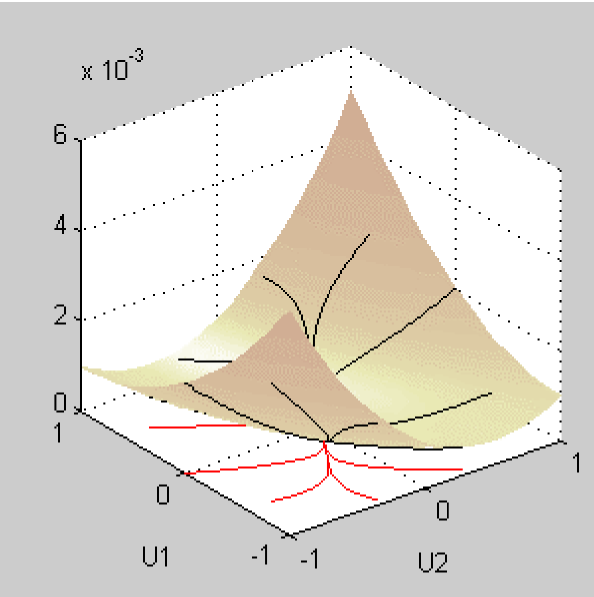

Fig. 2. Dissipative potential and virtual trajectory.

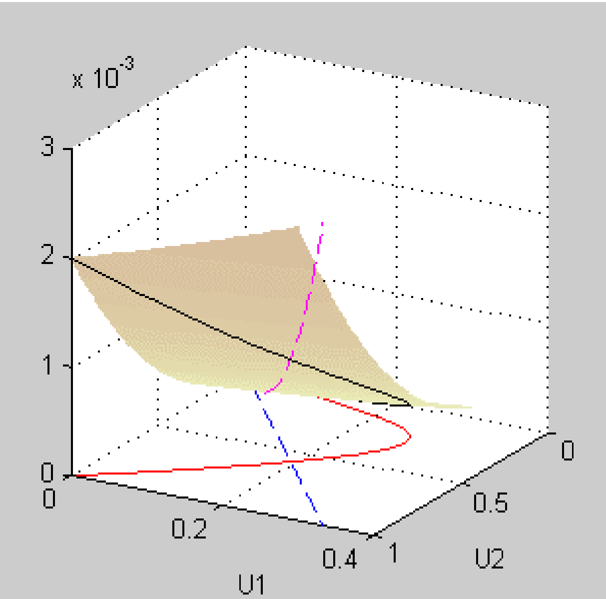

Fig. 3. Visualization of coupling (3).

The resulting trajectory will evidently follow the coupling line. Its direction is decisive for the stability, as illustrated in Fig. 4. From the original current $\vec{i} = -\nabla\Phi\Big|_{\delta\,\vec{u}}$, only its projection to the direction of the only possible motion (i.e. to the direction of coupling line) can play a role. The virtual trajectory will tend in the direction of this projection, i.e. towards the origin of coordinates. The given point is thus DC stable. The currents i_1 and i_2 are components of

coupling reaction i_r, which is perpendicular to the potential motion and which delivers no power.
The element in Fig. 1 (c) behaves as a „DC transformer“. It is characterized by remarkable features: It introduces no reciprocity into the circuit, and it does not cause DC instability. This element can be synthetized by current sources i_1 and i_2 which are derived from the basic current source λ according to the rule $i_1 = -A \cdot \lambda$, $i_2 = \lambda$. The current source λ will be adjusted to a proper value depending on the topology of resistive network.

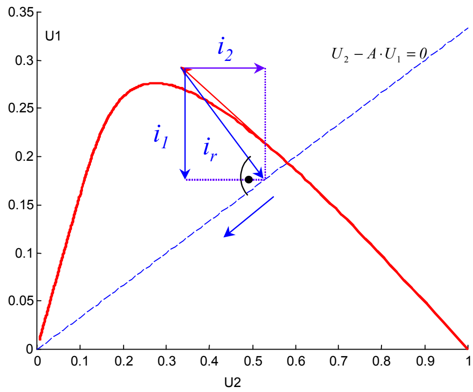

Fig. 4. Natural coupling reaction– DC stability

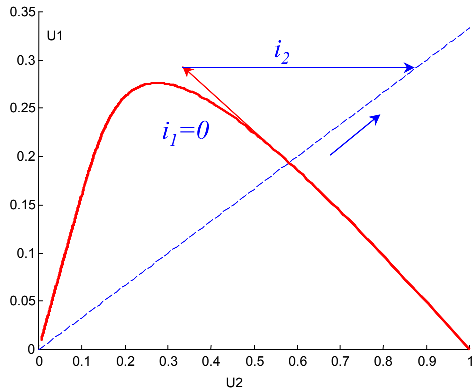

Fig. 5. Confined coupling reaction – DC instability

Fig. 5 shows the case where we try to fulfill condition (3) by utilizing a voltage amplifier with infinite input resistance, i.e. only by exciting node ②. Since the perpendicularity of reactive current to the given coupling cannot be guaranteed (it is not possible to affect the i_1 coordinate), the reaction delivers only virtual power to the circuit by current i_2, and the trajectory draws away from the equilibrium state. This point is DC unstable.
In this case, the cause of instability consists in the nonreciprocal realization of the required coupling condition. This is evident for the elements which ensure this condition by using only one outlet, as for instance in the case of OpAmp. The basic source of instability is that the system becomes *uncontrollable* under the given conditions (the control vector is incomplete).

Conclusion

The proposed method of DC instability testing is set so generally that it enables the investigation of a wide range of DC circuits. In particular, we focus on systems comprising elements with couplings of the type of “input-output response”, “N- port equations”, and elements which are described by the methods of behavioural modelling. The method described makes it possible to test systems with elements which have not been synthetized till now but whose required behaviour is known. In addition, the method can give directions for realizing such elements.

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