STABILITY PARADOX IN THE CIRCUITS WITH CONTROLLED SOURCES

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Abstract

Some paradox is described in this contribution consisting in the fact that in spite of evident circuit instability due to strong positive feedback, all poles of active dynamical circuit lie in the left-side complex plane. It is shown that the key problem is the incomplete circuit model, because the pole location can strongly depend on the fact if the parasitic reactances are included to the circuit model. This phenomenon is investigated together with so-called potential stability of DC operating point. Instability of the operating point results the instability of corresponding real dynamical circuit.

Introduction

Let us consider circuit in Fig.1. This circuit has single real pole which is moved along the real axis of complex plane in strange way due to change of gain *A*: Gain smaller than 1 results in the negative pole. This fact is in conformity with our knowledge that the circuit is stable because of weak positive feedback. Increasing gain above 1 causes jump of the pole to the positive real axis. However, following growing of the gain causes pole convergence to the axes origin and then again to the negative part of real axis. Pole is zero for the critical gain $A_0 = 1 + R_2/R_1$. We can see that utilization of classical stability criteria based on the pole location leads to the conclusion that the circuit is unstable for the gain from 1 to A_0 and that is stable for other values. In reality, investigated circuit is stable only for the gain $A<A_0$.

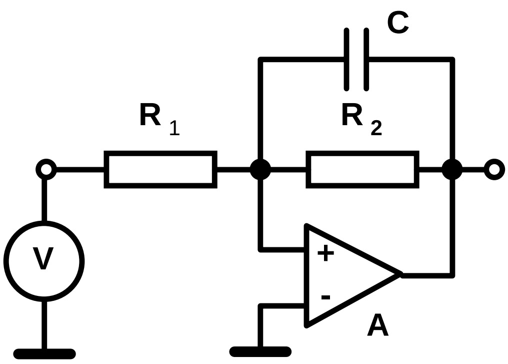

Fig.1.Example of circuit with controlled source.

Green showed in [1] that similar contradictions are based on the imperfect modeling of the dynamic properties of the investigated circuits. Some important results from [1] can be summarized to the following conclusions.

Stability of the dynamical circuits with controlled sources

<u>*Theorem 1:*</u> Some unstable/stable models of dynamical system with controlled sources can be done stable/unstable after inserting some set of <u>certain</u> positive-valued shunt capacitors and series inductors into the circuit.

<u>*Theorem 2:*</u> Some unstable/stable models of dynamical system with controlled sources can be done stable/unstable after inserting some set of <u>arbitrary</u> positive-valued shunt capacitors and series inductors into the circuit.

Taking into account the permanent presence of parasitic reactance elements, the aforementioned *Theorems* and especially second one are important for the understanding how the circuit with stable model in Fig.1 need not be stable in reality. *Theorem* 2 means that some places exist in the circuit under stability testing where the presence of reactance elements would be considered. In [1], the definition of such places is done:

Rule 1: Inputs of voltage-controlled sources and outputs of controlled current sources have to be closed by shunt capacitors.

Rule 2: Inputs of current-controlled sources and outputs of controlled voltage sources have to be in series with inductors.

For following purposes, let name circuit with measures mentioned in *rules* 1 and 2 as circuit with *inertia-type* controlled sources.

[1] consists further important idea:

Theorem 3: We complete resistive circuit with the necessary number of reactance elements to obtain circuit with inertia-type controlled sources. If this circuit is unstable, then it is not possible to do it stable by augmenting this circuit with an arbitrary set of shunt capacitors and series inductors. Then the operating point corresponding to the resistive circuit is declared as unstable.

We come to the idea of the stability of the DC operating point.

Stability of the resistive DC circuits with controlled sources

It is not easy to talk about the stability of pure resistive circuit because classical concepts are based on the observation of time evolution of circuit variables. However, this time evolution is not comprised in the algebraical equations describing resistive circuit because of the absence of time derivatives of circuit variables. For this reason, it is difficult to follow sequence of circuit phenomena or what is cause and what the implication. For example, if the input voltage of the VCVS (voltage controlled voltage source) which is connected to the resistive net grows immediately, then the following behaviour will depend on the fact if the output reaction will follow immediately or with certain delay. Second possibility is more practical because it takes the natural inertia of real systems into account. Here are the directions for understanding for the *Rules* 1 and 2.

Problem of the stability of DC resistive circuit can be transferred to the problem of the stability of its operating points. Operating point stability can be tested through the stability of corresponding dynamical circuit which is created using the linearization of resistive circuit around the operating point and using its augmenting with reactance elements.

If we consider controlled sources inside resistive circuit as *inertia-type* sources (see *Rules* 1 and 2), then the *Theorem* 3 results in the reality that if the circuit is unstable, it remains unstable after its augmenting with additional reactance elements. In other words, the operating point instability can be revealed only on the basis of algebraical equations describing DC resistive circuit. One procedure how to identify unstable operating point is described in [1]. However, this procedure sets only sufficient condition for operating point instability, but not a necessary one. In [2] we have published necessary condition which is based on the theory of so-called virtual dynamic of DC resistive networks.

Potential stability introduced in [1] is the opposite term to the instability of DC operating point. We try to define it more precisely then in [1].

Let us linearize DC resistive circuit around its DC operating point. The set of dynamical circuits can be attached to this linearized circuit by its augmenting with the shunt capacitors and series inductors. Utilizing all possibilities of this augmenting, the dynamical circuit is obtained which includes all possible attached dynamical circuits, taking into account various combinations of nonnegative values L and C of reactance elements. The set of all these circuits fills the space N of dynamical circuits attached to given DC operating point. Each of the dynamical circuits can be described by the vector D of the parameters L and C of its reactance elements. We define the norm $|D|$ of the vector D.

Let us consider one of the attached dynamical circuits described above. This circuit is described by the vector D_0. We define ε - neighbourhood of this circuit as the subspace of the space N containing this circuit and all attached circuits which vectors D fulfill condition

$$\left|D - D_0\right| \langle \ \varepsilon.$$

Definition 1: The DC operating point is potentially stable if two conditions are fulfilled at the same time:

1. The stable dynamical circuit exists in the space D.
2. There exists such $\varepsilon > 0$ such that all attached dynamical circuits inside the ε - neighbourhood of stable circuit are stable.

Definition 2: An operating point that is not potentially stable is said to be unstable.

It can be proved that the dynamical circuit from the item 1 of the *Definition* 1 is circuit with *inertia-type* controlled sources.

In [1], the definition 1 is formulated in more free way. First, the resistive circuit is augmented with resistive elements so that the stable circuit is formed. Corresponding operating point is potentially stable if any dynamic circuit created by additional augmenting with an arbitrary set of shunt capacitors and series inductors, whose values are sufficiently small, is also stable. First augmenting of resistive circuit need not be arbitrary but has to lead to the circuit with *inertia-type* controlled sources. Second augmenting then ensures that the definition is robust in the sense that additional attached stray reactance elements and their small variations do not change circuit's stability (see *Theorems* 1 and 2).

Paradox analysis

As shown in *Theorem* 3, the circuit with the incomplete model in Fig.1 will be unstable if operating point of corresponding resistive circuit will be also unstable. In this way, the stability analysis of our circuit can be performed through the stability testing of DC operating point. Because this circuit is linear, we can consider arbitrary operating point, for example zero-state one for zero input voltage. Starting DC resistive circuit is in Fig. 2a.

In Fig. 2b, the resistive circuit expansion to the circuit with *inertia-type* controlled sources is performed. Fig. 2c shows total expansion to the dynamical circuit from the space N. Each of the dynamical circuits is characterized by the vector

$$D = \left[C, L, C_1, C_2, L_1, L_2\right].$$

Obviously, the investigated circuit in Fig.1 belongs to the space N and its vector is

$$D = \left[0, 0, C_1, 0, 0, 0\right].$$

First two vector elements corresponding to the inertia of controlled source are zero. For this reason, stability analysis of the circuit from Fig.1 does not guarantee reliable results. On the other hand, if the instability will be detected by the analysis of circuit in Fig. 2b, we can be sure that both circuit in Fig. 2c and circuit in Fig. 1 as its special case will be unstable (see *Theorem* 3). In this way, using the analysis of circuit with inertia-type controlled source in Fig.2b the sufficient instability condition can be determined but not necessary one. Necessary

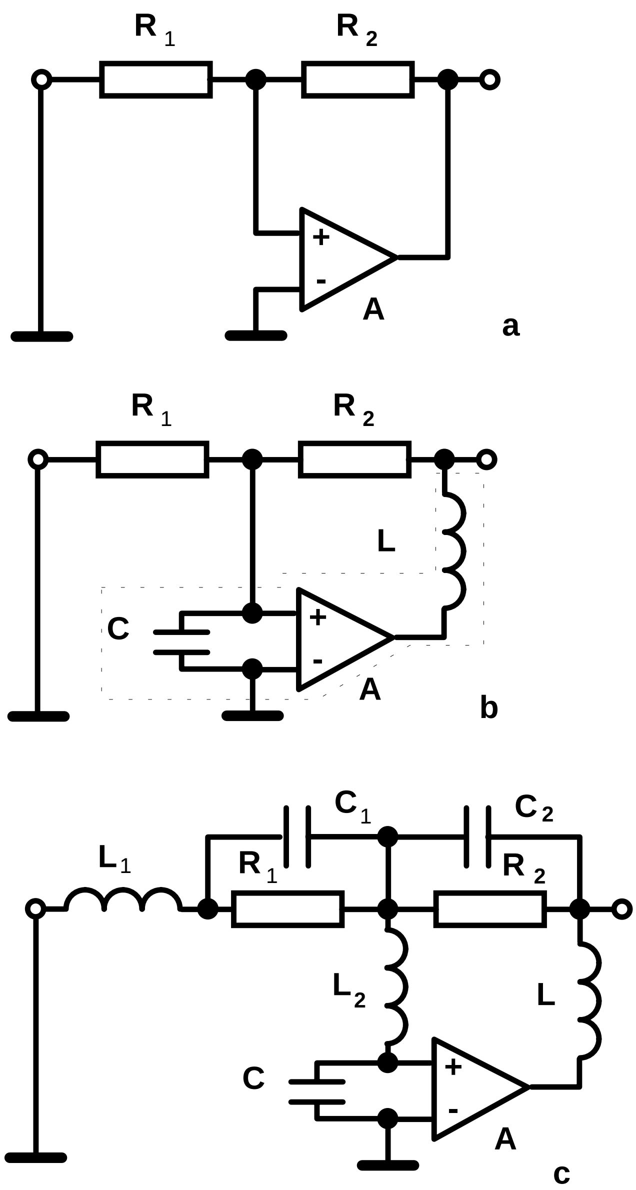

Fig.2.a) Starting resistive circuit,
b) corresponding circuit with *inertia-type* controlled source,
c) corresponding dynamical circuit from the space *N*.

condition would be obtained for instance by the analysis of the circuit in Fig. 2c in the sense of *Definition 1*. However, this procedure would be rather difficult.

Analysis of the circuit in Fig. 2b leads to the characteristic equation

$$s^2+\left(\frac{R_2}{L}+\frac{1}{R_1C}\right)s+\frac{1}{LC}\left(1-A+\frac{R_2}{R_1}\right)=0\,.$$

Circuit will be unstable for negative absolute term. We can write sufficient condition of operating point instability:

$$A>1+\frac{R_2}{R_1}.$$

We proved in [2] that this condition is also necessary.

Conclusion

During modeling of circuits with controlled sources, the influence of stray reactance elements upon the circuit stability has to be taken carefully into account. Inserting some reactance element to the certain places can course change of circuit stability. Due to influence of parasitic capacitances and inductances in the real circuits, there is necessary to consider reactance elements connected to these places. These places are inputs and outputs of controlled sources. We indicate connections with the stability of the DC operating point of corresponding resistive circuit including precise definition of the operating point potential stability. The potential stability criterion has been described in [2].

Acknowledgement

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References

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