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STABILITY TESTING OF DC CIRCUITS USING VARIATIONAL METHODS

Summary

The paper comprises considerations concerning some thought sources of circuit theory. It is shown that some return to the categories of analytical mechanics and nonequilibrium process thermodynamics can give a hand with the solution of such problems as the stability of DC circuits with nonreciprocal elements.

At first sight, analysis of circuits with controlled sources seems to be the closed part of circuit theory. However, putting the accustoming procedures into practice can lead to the paradoxical situations as shown for example in "school" problem with the independence of transfer function on the input polarity of operational amplifier.

In our paper, the roots of some paradoxical situations in circuit theory are investigated in more details. We investigate two restrictions of Newton's mechanics and analytical dynamics that have also influenced circuit theory.

1. INTRODUCTION

Analysis of circuits with controlled sources belongs to the well-known parts of circuit theory. However, putting the accustoming procedures into practice can lead to the paradoxical situations as shown in following "school" example.

Determine the voltage gain $F = V_2 / V_1$ according to Fig.1. Consider infinite input amplifier resistance and zero output resistance. Gain A is arbitrary nonzero real number.

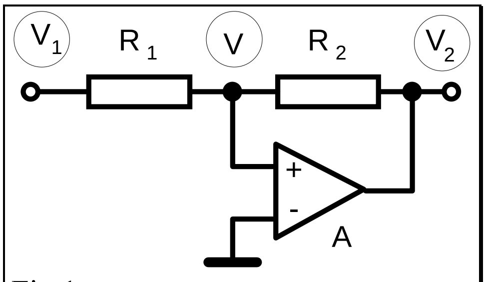

Fig.1

Utilization of arbitrary method based on Kirchhoff's laws and Ohm's law leads to the result

$$V_2 = F.V_1,\ \ F = -\frac{K_1}{K_2 - 1/A},\quad A \neq 0, \tag{1.1}$$

where $K_1 = \dfrac{R_2}{R_1 + R_2}$ and $K_2 = \dfrac{R_1}{R_1 + R_2}$.

It is remarkable that

$$\lim_{A\to\infty} F = \lim_{A\to-\infty} F = -\frac{R_2}{R_1}. \tag{1.2}$$

Equation (1.2) imposes the idea that the function of amplifier in Fig.1 does not depend on the polarity of input branch of OpAmp. However, this conclusion does not correspond to the practical experiences.

Paradox origin inside the thought system can point at certain restrictions of this system. For example, the well-known antique paradox about the Achilles and tortoise has arisen in consequence of the fact that the later knowledge of the area of infinitesimal calculus could not be included to the considerations. Next paradox - when the pile of stones stops to be the pile taking away stone by stone - stops be the paradox considering some principles of fuzzy sets. Our "OpAmp" paradox is caused due to peculiar position of stability in the circuit theory. It is conditioned historically because this doctrine has been under the influence of the Newton's mechanics and analytical dynamics. Equation (1.1) namely does not comprise information if the equilibrium point is stable or unstable. This equation only ensures that Kirchhoff's laws and coupling condition $V_2 = AV$ will be complied. There is no sense in definition of voltage gain F in case of unstable equilibrium point. In this way, the paradox (1.2) is explained.

It remains to investigate the stability conditions of our *DC* solution. As usual in circuit theory, the simple model in Fig.2 can be compiled using equations those have led to the result (1.1). The sign of open loop gain determines the stability, e.g.

$$A.K_2 < 0 \ \text{ or } \ A < 1 + \frac{R_2}{R_1}. \tag{1.3}$$

Let us explore the roots of the paradox (1.2) in more details. We investigate two restrictions of Newton's mechanics and analytical dynamics those have also influenced circuit theory. We concentrate our attention on the controlled sources from this rather unusual point of view.

2. NEWTON'S ASPECTS IN THE CIRCUIT THEORY

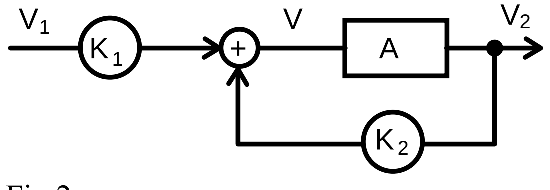

Fig.2

It has been proved during 50th years that Kirchhoff's voltage law (*KVL*) in the circuit theory corresponds to the 2nd Newton's postulate of the classical physics and consequently to the basic equation of variational calculus, i.e. Euler equation [1]. Choice of electrical charge q as generalized coordinates has been the assumption of this analogy. Then currents $i = \dot{q}$ have corresponded to generalized speeds, electrical voltages to generalized forces and magnetic fluxes to momentum. In this way, the ingenious tools of analytical dynamics could be used mainly for the analysis methods based on the *KVL*, e.g. for the little-known method of loop charges and for the method of loop currents.

It is less known that this principle is also utilized to the circuit analysis using nodal analysis method (*NAM*). Utilizing the dual properties of functions and cofunctions those describe electrical, magnetic and dissipative fields of used elements, the circuit **Lagrange function** can be constructed as a function of nodal voltages and their derivatives

$$L(\vec{v},\dot{\vec{v}}) = \overline{T} - V ,$$

where $\overline{T}(\dot{\vec{v}})$ and $V(\vec{v})$ are time derivatives of electrical field coenergy of charged capacitors and the magnetic field energy of inductors, $\dot{v}$ is the vector of nodal voltages.

In the area of *NAM*, the **Hamilton variational principle** can be formulated as follows:

The trajectory connecting points $\vec{v}(t_0)$ and $\vec{v}(t_1)$ that system chooses for its motion, minimizes the time integral of system Lagrange function

$$\int_{t_0}^{t_1} L(\vec{v},\dot{\vec{v}})dt \tag{2.1}$$

thus

$$\delta\int_{t_0}^{t_1} Ldt = 0 , \tag{2.2}$$

where δ means the symbol of variation of the whole trajectory between points $\vec{v}(t_0)$ and $\vec{v}(t_1)$ around the actual trajectory.

Euler equation is then equivalent to equation (2.2)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\vec{v}}}\right) - \frac{\partial L}{\partial \vec{v}} = \vec{0} . \tag{2.3}$$

The real trajectory $\vec{v}(t)$ is the solution of Euler equation for given initial conditions. The real trajectory is the extremale of variational task (2.2).

The circuit comprising also resistors and excited by current sources can be described using extended Euler equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\vec{v}}}\right) - \frac{\partial L}{\partial \vec{v}} + \frac{d}{dt}\left(\frac{\partial \overline{R}}{\partial \vec{v}}\right) = \dot{\vec{I}} \tag{2.4}$$

where $\dot{I}$ is the vector of current sources connected to circuit nodes,
$\overline{R}$ is the **Rayleigh dissipative cofunction** that maps the energy dissipation on resistors.

If the circuit is characterized by conductance matrix **G**, then

$$\overline{R}(\vec{v}) = \int_{\Gamma} \vec{I}^{T} \cdot d\vec{\dot{v}} = \int_{\Gamma} \left(\mathbf{G}\cdot\vec{\dot{v}}\right)^{T} \cdot d\vec{\dot{v}} \tag{2.5}$$

is the linie integral along the contour Γ from the origin of coordinates to the point $\dot{v}$. In case of circuits comprising linear reciprocal resistors, the Rayleigh function is as follows:

$$\overline{R}(\vec{v}) = \frac{1}{2}\vec{v}^{T}\cdot\mathbf{G}\cdot\vec{v} .$$

If the circuit contains the elements those set coupling of type $f(\vec{v},\dot{\vec{v}}) = 0$ (i.e. controlled sources, controlled switches etc.), this condition will be included to the Lagrange system function using the method of **indefinite coefficients** λ:

$$L^{*} = L + \lambda . f .$$

The physical interpretation of coefficient λ is the system response to the coupling *f*. This is the generalized force that the system has to expend to keep condition *f*. More information's can be found in the classical work [2].

If the circuit consists only of the resistive elements those are coupled by condition *f*, the simplified form of extended Euler equation may be used:

$$\frac{\partial(\overline{R}-\lambda f)}{\partial \vec{v}} = \vec{I} \ . \tag{2.6}$$

In case of linear reciprocal resistors we can write

$$\mathbf{G}.\vec{v} = \vec{I} + \frac{\partial(\lambda f)}{\partial \vec{v}} . \tag{2.7}$$

Neglecting coupling element, the equation (2.7) represents the basic equation of *NAM*.

We have seen that algorithms commonly used in the circuit theory are strongly influenced by the methods developed during past centuries in the area of classical physics. We have particularly followed the influence of first two Newton's postulates. However, the circuit theory has own peculiarities concerning **3rd Newton's postulate** of reaction.

3. POSTULATE OF REACTION

As mentioned in the opening example, the information about the circuit stability does not follow automatically the analysis method. This information must be obtained independently of the method additionally using special test. It looks to be something inferior not included to the main theory. This separation of the stability test and the rest analysis is typical not only for the circuit theory but also for many other branches like control theory. This is connected with the fact that these branches have assumed the conception of first two Newton's laws of classical mechanics but they have modified the third Newton's postulate of reaction according to own needs.

Control theory has modified this postulate to the form "reaction is equal to zero". This formulation has enabled to decompose originally two-way couplings in the system to the unilateral ones and to introduce the system as orientated "block" diagram (the sample of such diagram consisting exclusively of the elements with the unilateral forward coupling is in Fig.2). More information can be found in [3].

Circuit theory let the reaction character to the nature of individual elements that can be either reciprocal (action = reaction) or nonreciprocal (action ≠ reaction).

Due to modification of the postulate of reaction, various theories arise. Some their conclusions seem paradoxically from the classical physics point of view. For instance, the perpeetum mobile can exist in the circuit and control theory (oscillator etc.). It is interesting for our suggestion that invalidity of the law of energy conservation is one of the implications of the 3rd Newton's postulate modification. This fact is practically projected to the possibility of system instability. This possibility then follows naturally the axiomatic of given theory. Here there is necessary to seek explanation of why the stability test seems to be alien element in the area of such theory.

Let us then try to analyze our circuit in Fig.1 from the positions mentioned in chapter 2. Fig.3 shows the same circuit prepared for the *NAM* description. The input voltage source has been replaced by the current source $I_1 = V_1 G_1$. Applying equation (2.7) yields

$$\begin{bmatrix} G_1+G_2 & -G_2 \\ -G_2 & G_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \end{bmatrix} + \begin{bmatrix} -A.\lambda \\ \lambda \end{bmatrix} . \tag{3.1}$$

Equilibrium in the resistive network with the coupling condition

$$f = V_2 - AV_1 = 0 \tag{3.2}$$

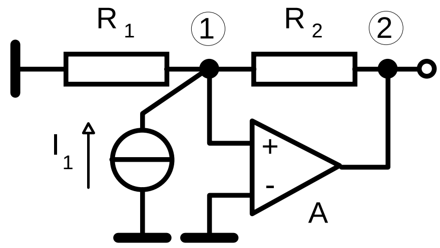

Fig.3

is reached by means of compensation current sources affecting both nodes 1 and ②. On the other hand, it is known that ideal voltage amplifier ensures condition (3.2) only using its output, i.e. affecting only node ②. This contradiction has arose because of the regulation (2.7) comes out the validity of the Newton's postulate of the equality of action and reaction. However, the voltage amplifier as nonreciprocal element violates this regularity by its own law that "reaction is equal to zero".

Taking into account aforementioned facts, equation (2.7) can be generalized as follows

$$\mathbf{G}.\vec{v} = \vec{I} + \mathbf{K} \cdot \frac{\partial(\lambda f)}{\partial \vec{v}} \tag{3.3}$$

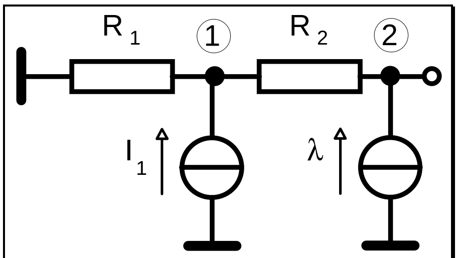

Fig.4

where **K** is the matrix of weight coefficients that adapts relation between the action and reaction according to reality.

Let us then insert information that our voltage amplifier has infinite input resistance to equation (3.3). We obtain

$$\mathbf{G} \cdot \vec{v} = \begin{bmatrix} I_1 \\ \lambda \end{bmatrix}. \tag{3.4}$$

Meaning of the undetermined coefficient λ is then the current supplied to the network by the output of amplifier. Equation (3.4) along with the condition (3.2) give directions for computation of both nodal voltages and unknown current λ.

Now let us investigate how to test stability of circuits without energy storage elements using variational principles.

4. PROBLEM OF ENERGY DISSIPATION

If the system has no storage elements, then the most well-known criterion cannot be used to stability test because the time factor is not entirely presented. For instance, this problem is solved in [4] in respect of the algorithms for computation of *DC* operating points in the simulation program *SPICE*. The term "*potential stability*" is introduced in this work. The presented algorithms operate in this way that parasitic inductances or capacitances are added to the circuit with the aim to reach certain dynamics.

As shown in aforementioned chapter, the alone possibility of state instability is paradox as a matter of fact which is caused by the modification of the 3rd Newton's postulate of reaction. As regards circuit without inertia, this problem is still more complicated because alone Newton's dynamics is originally the doctrine about conservative systems. Resistor as the element dissipating energy into heat is the alien element in the theory based on the Newton's conception. This fact is well evident in the extended Euler's equation (2.4): the Rayleigh's function is violently placed here to express real resistors. Resulting trajectory of such system is not extremale of variational principle (2.2). Hamilton's principle is in its essence valid only in case of conservative systems. This is the sign of the period in which the problems of celestial mechanics and other phenomena were solved. In these phenomena the system dynamics played dominant role in comparison with the comparatively negligible dissipation of kinetic energy.

However, ideal resistor as an element generating heat has became the central point of relatively young doctrine - nonequilibrium thermodynamics. This doctrine also reports to the terms as capacitor and inductor, gives directions for the compilation of system motion equations and it also has its own variational principles those mostly correspond to the principles of analytical dynamics. The nonequilibrium thermodynamics reveals that the integral Hamilton's principle cannot be reformed for the systems with dissipative elements as resistors those thermodynamic essence baffles Newton's conception.

It is possible to search answers to the stability problems of pure dissipative systems in the *thermodynamical* variational principles. Among them, the differential principles dominate, i.e. principles referring to the instantaneous system behavior in the concrete trajectory point. One of the form of least energy dissipation principle given by *Glansdorff* and *Prigogine* [5] is suitable for the circuit description using *NAM*. This principle can be loosely formulated as follows:

During motion, system minimizes the expression

$$\Phi(\vec{v}) = \overline{R}(\vec{v}) - \vec{I}^{\,T}.\vec{v} \tag{4.1}$$

in view of the possible instantaneous variations of nodal voltages.

Function $\overline{R}$ is the Rayleigh's system cofunction defined according to (2.5).

This principle can be formulated in more practical way:

Stable *DC* operating point occurs in the local minimum of function (4.1).

Remark: As regards nonreciprocal systems, the choice of the contour Γ of integration is important for the Rayleigh cofunction definition (2.5).

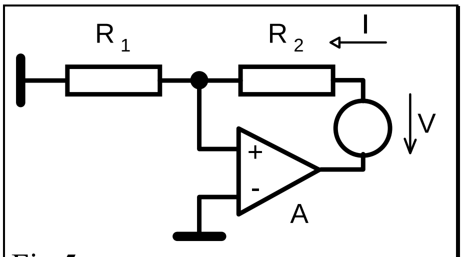

Fig.5

Let us attempt to analyze stability of our voltage amplifier circuit using the function Φ. After disconnection of outside excitation to node ① according to Fig.3, we can only construct Rayleigh cofunction using (4.1).

We disconnect output of amplifier. Our task is to search function $\overline{R}$ of circuit with open feedback loop (see Fig.5):

$$\overline{R}(V) = \int_0^V I(\tilde{v})d\tilde{v}\ .$$

It can be easily proved that $I = V/R$ where $R = R_1 + R_2 - AR_1$. Thus $\overline{R}(V) = V^2/2R$. If $R > 0$ then Rayleigh cofunction has local minimum in the operating point $V = 0$. Then $A < 1 + R_2/R_1$. However, this is the relation (1.3) derived in the first chapter.

5. CONCLUSION

Equation (2.3) shows that *KCL* and Euler's equation of variational calculus are equivalent if the suitably chosen circuit voltages are selected as system coordinates. This fact enables to use variational methods for the circuit analysis using *NAM*. Equation (3.3) is universal. For instance, *DC* circuits with nonideal OpAmps those models are given using conditions $f_i(\vec{v}, \dot{\vec{v}}) = 0$ can be analyzed using this equation. Thermodynamical stability criterion (4.1) can be also used for the stability testing of large circuits with nonreciprocal elements as OpAmps. The suitable selection of integration contour for the construction of Rayleigh cofunction is important key to success.

REFERENCES

[1] MYSLÍK,J.: Analysis of electrical circuits using variational methods. Electrical Engineering Journal, No.4, 1972. Czechoslovakia.

[2] LANCZOS,C.: The variational principles of mechanics. Mathematical expositions, No.4, Toronto Press, 1962.

[3] GALPERIN,I.I.: Automatics as one-sided mechanics. Moskva 1964.

[4] GREEN,M.M.-WILLSON,A.N.: How to Identify Unstable DC Operating Points. IEEE Trans. on CAS I, vol. 39, No.10, 1992, pp.820 - 832.

[5] GYARMATI,I.: Non-equilibrium thermodynamics. Springer-Verlag, New York 1970.

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