NEW STABILITY ALGORITHM FOR CIRCUITS WITH CONTROLLED SOURCES

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ABSTRACT

A new method of stability testing of dynamical circuits linearized around dc operating points is proposed. In many cases, classical "pole-location" methods may fail. The procedure is based on the energetic mechanisms inside the circuit that is deflected from given equilibrium state due to fluctuation. As a result, the general stability criterion is formulated including the new computer algorithm.

INTRODUCTION

This paper extends the ideas described in [1], where the so-called fluctuation theory of stability is introduced. We summarize the main results.

Consider a resistive circuit and its investigated operating point. Some fluctuation causes a small deflection from the operating point. It is shown that for a stable operating point, the energy of fluctuation is fully consumed by the circuit – the circuit behaves like a consumer. In case of unstable operating point, the circuit behaves like a source and the original fluctuation develops into unstable behavior.

The so-called virtual eigenvalues of resistive circuit have been defined to model the phenomenon described above. Their number is given by the number of controlled sources in the linear model of tested circuit. The operating point stability can be tested using the eigenvalues location: in case of stability, the real parts of all virtual eigenvalues have to be positive.

The criterion described is also generalized to the dynamical circuits, where the virtual eigenvalues depend on frequency. The given trajectory in the complex plane corresponds to each eigenvalue for the frequency range $\omega \in \langle 0, \infty)$. In case of unstable equilibrium state, such a frequency has to exist when the pure active power is transferred from the circuit to the source of fluctuation. As a practical rule of instability, there is at least one trajectory, which touches the negative real half-axis of the complex plane.

This rule has the character of graphical method like Nyquist's stability criterion. Its practical utilization is not convenient for large systems. In the following chapter, we propose a method that provides for algorithmic decision about the stability.

METHOD

Consider a nonlinear dynamical system that is linearized round the investigated operating point. As a result, a linear N_d - order dynamical system with N_s controlled sources is obtained. Zero voltages/currents of controlled sources correspond to the dc operating point.

Fluctuations round the equilibrium state can be modeled by additional sources of fluctuations. We add a voltage source of fluctuation δV_i in series to each controlled voltage source V_i. Similarly, we add a current source of fluctuation δI_i in parallel with each controlled current source I_i. The voltage and current fluctuations can be arranged into the vector $\delta\mathbf{X}$:

$$\delta\mathbf{X} = [\delta V_1\ \delta V_2 \ldots \delta I_1\ \delta I_2 \ldots]^T = [\delta\mathbf{V}\ \delta\mathbf{I}]^T .$$

The circuit reactions to these fluctuations are the currents δI_{Ri} of voltage fluctuation sources and the voltages δV_{Rj} of current fluctuation sources (we respect the source orientation of voltages and currents). Due to linearity, reaction is proportional to action:

$$\delta\mathbf{X}_R = \mathbf{H}\delta\mathbf{X}, \qquad (1)$$

where

$$\delta\mathbf{X}_R = [\delta I_{R1}\ \delta I_{R2} \ldots \delta V_{R1}\ \delta V_{R2} \ldots]^T = [\delta\mathbf{I}_R\ \delta\mathbf{V}_R]^T$$

is the vector of reactions and **H** is the so-called fluctuation matrix.

Taking into consideration the spectral properties of fluctuations and of the investigated system, the vectors $\delta\mathbf{X}$, $\delta\mathbf{X}_R$, and the fluctuation matrix **H** are frequency dependent. Formally, the substitution $j\omega = s$ yields

$$\delta\mathbf{X}_R(s) = \mathbf{H}(s)\delta\mathbf{X}(s). \qquad (2)$$

The following statements are true:

Statement 1:
All eigenvalues

$$\lambda_i(s), \quad i = 1..N_s$$

of the fluctuation matrix $\mathbf{H}(s)$ are rational fractional functions of the operator s.

Statement 2:
The equilibrium state is stable if:

- all eigenvalues of the fluctuation matrix $\mathbf{H}(s)$ for $s = 0$ lie in the right-half complex plane, and

- all poles of all eigenvalues of the fluctuation matrix $\mathbf{H}(s)$ lie in the left-half complex plane.

Statement 3:
The poles of all eigenvalues of the fluctuation matrix $\mathbf{H}(s)$ and the classical poles of the investigated dynamical system are identical.

The Statements 2 and 3 yield following conclusion:

Statement 4:
The equilibrium state is stable if:
- the corresponding dc operating point is stable, and
- the classical stability test of the dynamical circuit based on the pole-location methods is positive.

Explanation:
In case of simple eigenvalues of matrix $\mathbf{H}$, we can find such linear transformations

$$\begin{aligned}\delta\mathbf{Y}_R &= \mathbf{P}^{-1}.\delta\mathbf{X}_R,\\ \delta\mathbf{X} &= \mathbf{P}.\delta\mathbf{Y}\end{aligned} \tag{3}$$

which transform equation (2) into other voltage and current coordinates:

$$\delta\mathbf{Y}_R(s) = \Lambda(s).\delta\mathbf{Y}(s), \tag{4}$$

where

$$\Lambda = \mathbf{P}^{-1}.\mathbf{H}.\mathbf{P} = \boxed{\begin{matrix}\lambda_1 & & 0\\ & \ddots & \\ 0 & & \lambda_{Ns}\end{matrix}} \tag{5}$$

is the Jordan diagonal matrix with the same eigenvalues $\lambda_1,..., \lambda_{Ns}$ as for the original matrix $\mathbf{H}$.
As a consequence of the diagonality, equation (5) is the set of N_s scalar equations

$$\delta Y_{Ri}(s) = \lambda_i(s).\delta Y_i(s), \quad i = 1..N_s\,.$$

Here $\delta Y_{Ri}(s)$ and $\delta Y_i(s)$ are the Laplace transforms of the variables of the transformed linear dynamical system. In this way, the eigenvalue $\lambda_i(s)$ is a circuit function that is in the classical form for circuits with lumped parameters, i.e. the rational fractional function of the complex frequency s. In this way, Statement 1 is explained.

For $s = 0$, equation (2) is simplified to the fluctuation equation of resistive system. As shown in [1], the eigenvalues of the fluctuation matrix then describe the stability of a dc operating point. As proved in [1], all eigenvalues have to lie in the right-half of the complex plane for the stable dc operating point. The stable dc operating point is the necessary but not sufficient condition of the stability of corresponding equilibrium point of the adjoined dynamical system. This is the explanation of the first part of Statement 2.

For $s \neq 0$, the frequency properties of the fluctuations are considered. For the stable equilibrium states, the substitution $s = j\omega$ leads to the fluctuation equation

$$\delta\mathbf{X}_R(j\omega) = \mathbf{H}(j\omega)\delta\mathbf{X}(j\omega)$$

with the following physical interpretation: the system responds to the fluctuations round the equilibrium state as a filter with the matrix frequency response $\mathbf{H}(j\omega)$.

For a stable reaction to the fluctuation, all poles of all eigenvalues $\lambda_i(s)$ have to lie in the left-half complex plane.

The equation (1) can be rewritten to the form

$$\underbrace{\begin{bmatrix}\delta\mathbf{I}_R\\ \delta\mathbf{V}_R\end{bmatrix}}_{\delta\mathbf{X}_R} = \underbrace{\begin{bmatrix}\mathbf{G} & \mathbf{B}\\ \mathbf{A} & \mathbf{Z}\end{bmatrix}}_{\mathbf{H}}\underbrace{\begin{bmatrix}\delta\mathbf{V}\\ \delta\mathbf{I}\end{bmatrix}}_{\delta\mathbf{X}}. \tag{6}$$

From the two-port terminology, the corresponding two-port coefficients are in the submatrices $\mathbf{G}$, $\mathbf{B}$, $\mathbf{A}$, $\mathbf{Z}$, on the assumption that an input/output two-port variables are in the $\delta\mathbf{X}/\delta\mathbf{X}_R$ vectors.

In this way, the elements of the matrix $\mathbf{H}$ are a rational fraction functions of Laplace operator s, where all denominators are given by the characteristic polynomial of the investigated dynamical system. After the linear transformation (5), these denominators are not changed. That is why the Statement 3 is true.

The Statement 4 yields a direction how to test the stability: the classical test based on the pole-location has to be completed by the testing of the stability of dc operating point. For the stable system, both tests have to be positive.

ALGORITHM SUMMARY

1. *We linearize a nonlinear system round a given operating point. As a result, a linear model with controlled sources is obtained.*
2. *We add a voltage source of fluctuation δV_i in series to each controlled voltage source V_i.*
3. *We add a current source of fluctuation δI_i in parallel with each controlled current source I_i.*
4. *We mark circuit reactions to the fluctuations: currents δI_{Ri} of voltage fluctuation sources and voltages δV_{Rj} of current fluctuation sources (we respect the source orientation of voltages and currents).*

5. *We compile the fluctuation matrix **H**(s=0) as simplified relation (2) between the source and the response fluctuations.*
6. *We compute all eigenvalues of the matrix **H** for s = 0. If at least one of them has a negative real part, the corresponding dc operating point is unstable. Then the behavior of tested system is also unstable. The testing procedure can be stopped. Otherwise, we continue to the item 7.*
7. *We perform the classical pole-location stability testing of the corresponding dynamical system. If at least one of the poles has a positive real part, or some multiple poles in the imaginary axis appear, the corresponding equilibrium point is unstable. Then the behavior of tested system is also unstable. Otherwise, the equilibrium point is stable.*

EXAMPLE

Consider the low-pass filter in Fig. 1 (the cutoff-frequency 3.4 kHz, pass-band ripple 0,2 dB, attenuation at least 30 dB above the frequency 4 kHz, group delay max. 1msec.). Both OpAmps are simple types 741. In the course of the stability testing, the verification of the polarity of input OpAmps outlets can be also performed.

<u>*OpAmps polarity as in Fig. 1*</u>.

For the testing, we add the voltage sources of fluctuation δV_1 and δV_2 in series to the outputs of *OPA*1 and *OPA*2. The circuit reactions are in the form of the current fluctuations δI_{R1} and δI_{R2}. For the dc operating point, equation (1) is as follows:

$$\begin{bmatrix} \delta I_{R1} \\ \delta I_{R2} \end{bmatrix} = \underbrace{\begin{bmatrix} 5{,}91710.10^{-11} & -3{,}80843.10^{-16} \\ -3{,}80843.10^{-16} & 1{,}38118.10^{-10} \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} \delta V_1 \\ \delta V_2 \end{bmatrix}$$

The fluctuation matrix **H** has two eigenvalues

$$\lambda_1 = 5{,}91722.10^{-11},\ \lambda_2 = 1{,}3811821.10^{-10}.$$

Both eigenvalues are positive. That is why the corresponding dc operating point is stable.

Now let us compute the classical poles of investigated filter. The pole-location depends on the OpAmps model. First consider simple model in the form of VCVS with the frequency independent voltage gain A_0 = 200000 (corresponds to the type 741). Then the filter has 5 poles:

$$-1{,}43752.10^3 \pm j2{,}20327.10^4$$

$$-7{,}01390.10^3 \pm j1{,}79214.10^4$$

$$-1{,}423917.10^4.$$

Now consider more complex one-pole model, which includes the open-loop gain A_0 = 200000, GBW = 1MHz and the output resistance $R_O = 75\Omega$. In this case, two additional poles appear. The resulting pole-location is then as follows:

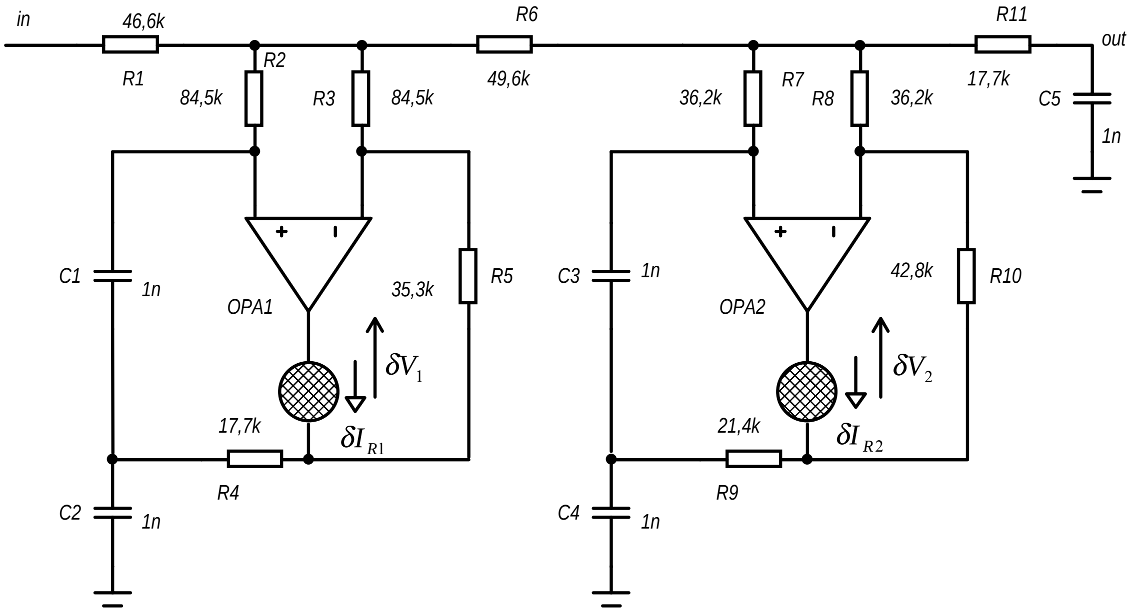

Fig.1. Active filter under the test.

$-1{,}38364.10^3 \pm j2{,}18414.10^4$

$-6{,}99813.10^3 \pm j1{,}76378.10^4$

$-1{,}43004.10^4$

$-3{,}336247.10^6$

$-4{,}75325.10^6$.

In both cases, all poles lie in the left-half complex plane. In accordance with the Statement 4, the investigated equilibrium state of the filter is stable.

Interchanged polarity of the OpAmps.

Let us interchange the input outlets of the first amplifier *OPA*1. It should be noted then the circuit will be unstable.

Compilation of the equation 1 leads to the result

$$\begin{bmatrix} \delta I_{R1} \\ \delta I_{R2} \end{bmatrix} = \underbrace{\begin{bmatrix} -5{,}91722.10^{-11} & 3{,}80850.10^{-16} \\ 3{,}80843.10^{-16} & 1{,}38118.10^{-10} \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} \delta V_1 \\ \delta V_2 \end{bmatrix}$$

One of the eigenvalues of the matrix **H** is negative:

$$\lambda_1 = -5{,}91722.10^{-11},\ \lambda_2 = 1{,}3811821.10^{-10},$$

and the dc operating point is unstable. The classical pole-location need not be computed – the behavior of investigated filter is unstable.

Nevertheless, let us do it. The simplified VCVS-OpAmps model yields following 5 poles:

$-1{,}43752.10^3 \pm j2{,}20325.10^4$

$-7{,}01382.10^3 \pm j1{,}79216.10^4$

$-1{,}423920.10^4$.

In spite of the circuit instability, all poles lie in the left-half complex plane!

Now consider one-pole OpAmps model mentioned above. One positive pole appears:

$-1{,}26216.10^3 \pm j2{,}21563.10^4$

$-7{,}10393.10^3 \pm j1{,}76489.10^4$

$-1{,}42865.10^4$

$-3{,}33935.10^6$

$\mathbf{+4{,}59869.10^6}$.

Obviously, the result of the classical stability testing depends on how to model real properties of circuit elements. This fact can cause the testing results to be unreliable, especially in case of large transistor structures.

If the testing of the dc operating point stability precedes, these improper phenomena are excluded.

To confirm aforementioned conclusions, similar analysis could be done for the inputs interchanging of the second amplifier *OPA*2 or both amplifiers.

CONCLUSION

A new algorithm how to test the stability of nonlinear circuits round given operating points is proposed. Our method is based on the so-called fluctuation theory of stability. The above method yields correct results in the cases where the classical "pole-location" methods can fail (e.g. circuits with controlled sources), and where no general tools are known (e.g. stability of dc operating points of resistive circuits).

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REFERENCES

[1] BIOLEK,D.-BIOLEK,Z.: Stability Of Dc Operating Points And Corresponding Dynamical Systems. AMSE, MS'97, Melbourne, pp. 236-240.

[2] GREEN, M.M.-WILLSON, A.N.: How to Identify Unstable dc Operating Points. IEEE Transactions on Circuit and Systems, Vol.39, No.10, October 1992.

[3] GREEN,M.M.: (Almost) Half of Any Circuit's Operating Points are Unstable. IEEE Trans. on Circuit and Systems, Vol. 41, No.4, April 1994, pp. 286-293.

[4] BIOLEK,D.-BIOLEK,Z.: Potential Stability of DC Operating Points in the Circuits with Controlled Sources. ICECS'96, Rodos, Greece, Vol. I, pp. 101-104.

[5] BIOLEK,D.-BIOLEK,Z.: Virtual Dynamics of DC Circuits. MWSCAS'96, Ames, Iowa, USA, Vol. II, pp. 951-954.

[6] BIOLEK,Z.-BIOLEK,D.: Stability of DC Operating Points in the Networks with Controlled Sources. SYS'95, AMSE, July 3-5, 1995, Brno, Vol.4, pp. 26-30.