

# IMITTANCE STABILITY CRITERION

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**Abstract:** In this contribution, we discuss a frequency equivalent of the method of negative resistances for the stability testing of equilibrium points in electrical circuits.

## 1. Introduction

The so-called DC stability or the stability of DC operating point is discussed in [1]. The main ideas are as follows:

1. The nonlinear circuit is linearized around an investigated operating point. The linear resistive circuit with controlled sources is obtained.
2. A DC circuit's operating point is said to be potentially stable if, by inserting some set of positive-valued shunt capacitors and series inductors into the linear resistive circuit, the corresponding equilibrium point of the resulting dynamic circuit is stable, and if the equilibrium point of any dynamic circuit created by augmenting this dynamic circuit with an arbitrary set of shunt capacitors and series inductors, whose values are sufficiently small, is also stable.
3. An operating point that is not potentially stable is said to be unstable. In this case, the circuit cannot be stabilized any more.

The DC stability can be investigated using the method of negative resistances [2]. We measure the differential resistance between any two nodes and between any two points arisen by cutting arbitrary branch. If some negative resistance is found, given equilibrium point is DC unstable. The **DC stability is the circuit ability to eliminate slow fluctuations round the equilibrium point**, which can be caused e.g. by the power supply or temperature fluctuation, gradual changing of the component parameters, etc. If the system trajectory approaches such DC stable point after the switching the supply on, it is attracted to this point which becomes the operating point. On the contrary, the DC unstable point cannot become the observable operating point.

A speed field is illustrated in Fig. 1, which gives the operating point deflected by a fluctuation back to the original equilibrium point. As shown, that more closely the equilibrium point the weaker field. In fact, it is the “top” view of the dissipative depletion region [3]. However, this figure is only valid for the infinitesimally slow fluctuations. In such cases, it is proper to short all inductors and disconnect all capacitors, and do not worry about the parasitic reactances.

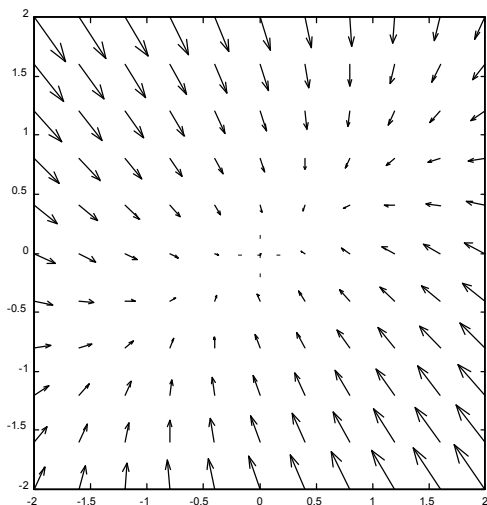


Fig. 1. A speed field round the operating point

In reality, the fluctuations are wideband. For the fast changes, the system can escape easily from the depletion region and it becomes unstable.

The essence of the method of negative resistances consists in the searching such feedback paths, which operate in case of very *slow* fluctuations due to negative differential resistances. The so-called virtual eigenvalues of the corresponding resistive circuit [4] then represent the dominant resistances/conductances, to these the fluctuations work. However, the real fluctuations work to the circuit impedances/admittances which are frequency dependent. The general question is as follows: is it possible to decide about the operating point stability from their frequency characteristic?

## 2. Imittance criterion

The question mentioned above could be reworded:

Consider the voltage or current fluctuation, which works to an imittance  $X(j\omega)$ ,  $\omega \in \langle 0, \infty \rangle$ . Knowing this frequency dependence, is it principally possible to find out the circuit stability in face of this fluctuation?

Before giving the solution, let us introduce one approach based on the Laplace transform of the imittance:

The equilibrium point is stable if and only if the loading imittance  $X(s)$  of an arbitrary source of the fluctuation fulfills the condition

$$\lim_{t \rightarrow \infty} [Laplace^{-1}\{X(s)\}] = \lim_{t \rightarrow \infty} x(t) = 0, \quad (1)$$

where  $x(t)$  is the corresponding circuit response to the fluctuation in the form of the unity step.

Given definition starts from the well-known stability conception: For the stable system, the current/voltage reaction to the voltage/current impulse must tend to zero.

It is necessary to use Laplace (not Fourier) transform, because the impulse responses of unstable systems are not absolutely integrateable. Nevertheless, there exists a frequency analogy of the criterion (1).

The condition (1) is fulfilled if the imittance  $X(s)$  has no poles in the right-side complex plane. The following frequency criterion can be derived using the theory of the functions of complex variables:

Be  $X(s)$  a loading imittance of the fluctuation source. The equilibrium point is stable regarding this fluctuation if the above conditions will be fulfilled at the same time:

- $\lim_{\omega \rightarrow 0} Re\{X(j\omega)\} > 0$ ,
- the complex plot  $X(j\omega)$  for  $\omega \in \langle 0, \infty \rangle$  circles clockwise the origin of the complex plane even  $N$ -times, where

$N =$  number of zeros of the imittance  $X(s)$  in the right-side complex plane.

The first condition ensures the DC stability, the second one is an analogy of the Nyquist's stability criterion for the imittances. In case of the voltage/current fluctuation, we must examine the complex plot of the admittance/impedance.

In most cases, the imittance has no zeros. Then this criterion is reduced to the following: the imittance complex plot must not intersect the negative real axis.

This criterion has an interesting physical meaning. Some details will be given in a special paper.

## 3. Application

Consider the voltage amplifier in Fig. 2. The given operating point, which is found by the computer simulator, is unstable. Not even the capacitance  $C$  can change this attribute. If you do not consider the OpAmp finite bandwidth, serious problems appear.

Without the amplifier, the circuit is described by the differential equation

$$\tau \cdot \dot{v}_1(t) + v_1(t) = \tau \cdot \dot{v}_2(t) + k \cdot v_2(t),$$

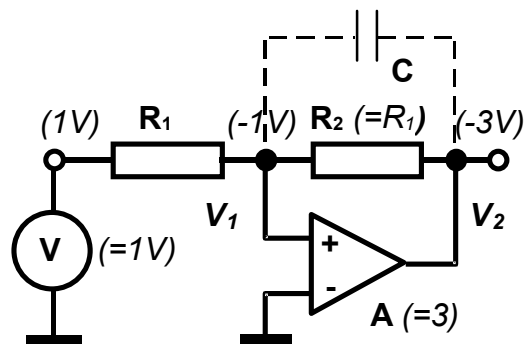


Fig. 2. DC unstable equilibrium point.

where  $\tau = \frac{R_1 R_2}{R_1 + R_2} \cdot C$ ,  $k = \frac{R_1}{R_1 + R_2}$ .

Taking into account the coupling condition  $v_2(t) = A \cdot v_1(t)$ , which brings the amplifier into the circuit, the following equation can be derived:

$$\tau \frac{1-A}{1-kA} \cdot \dot{v}_1(t) + v_1(t) = 0. \quad (3)$$

However, this equation offers the absurd conclusions. For example, in case of big positive  $A$ , the solution of (3) is stable in spite of the reality. Our approach gives the negative loading output resistance. The instability is ratified.

The classical stability approaches have failed due to “neglecting” of the amplifier dynamics. Let us correct it now. In addition, assume a nonzero amplifier input-output delay  $T_d$ . We choose a tiny value 10psec. The resistances and capacitances be 1k $\Omega$  and 1nF. The resulting complex plots of the loading output admittance for various amplifier gains and bandwidth are illustrated in Fig. 3. These plots are constructed for the frequency range  $\omega \in (0, \infty)$ .

The analysis of the given results leads to the interesting conclusion: The stability for the gain  $A \in (-\infty, -1)$  is ensured by the limited bandwidth of the amplification path, not by the existence of the “negative feedback”, as often mentioned. For  $A \in (1, \infty)$ , trajectories of the loading admittance start on the negative real axis. That is why the circuit is DC unstable.

#### 4. Conclusion

As shown in this article, the DC stability of the linearized circuits can be tested on a basis of the frequency dependence of the loading imittances, to which the ever-presented fluctuations work. To solve this problem, the imittance variant of well-known Nyquist’s stability criterion is formulated.

#### References:

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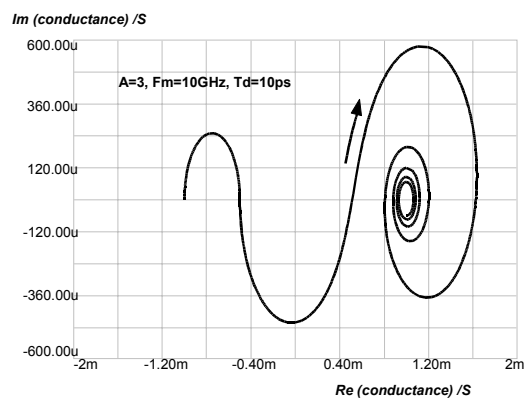
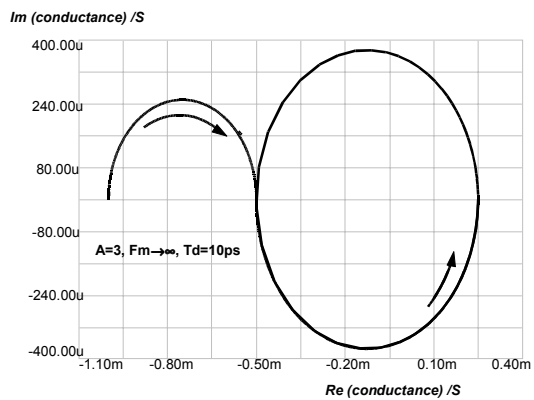
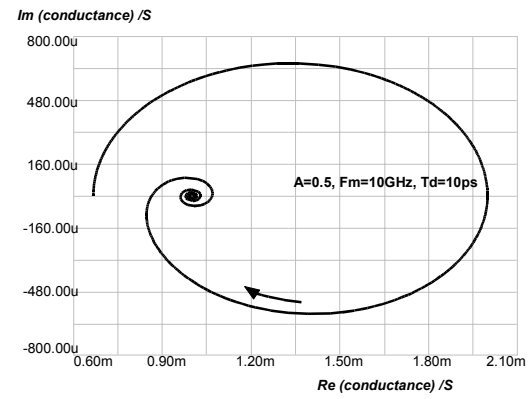
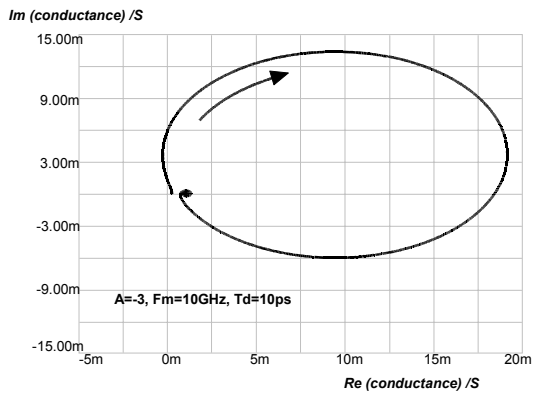
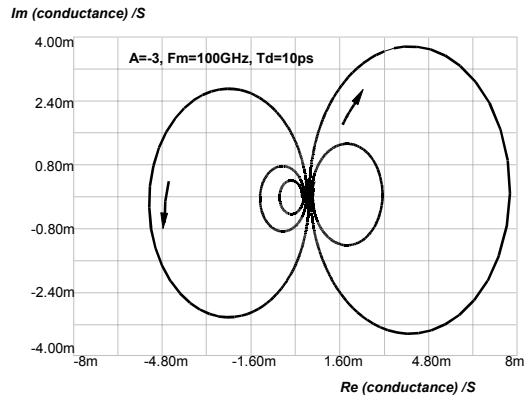
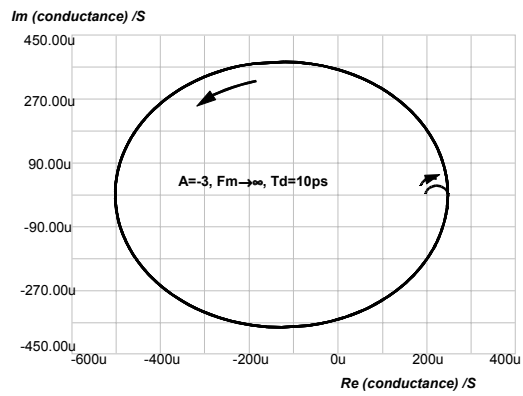


Fig. 3. Trajectories of the loading admittances (circuit in Fig. 2).